7.9—Proving a Quadrilateral is a Rhombus, Rectangle, or a Square

7.9 Day 1 Warm Up
For each figure, tell if there is enough information to prove that the quadrilateral is a parallelogram. If so, give the theorem or definition.

1. 
2. 

3. \( m\angle A + m\angle D = 180^\circ \) 
4. 

\[ \begin{align*} m\angle A + m\angle D &= 180^\circ \end{align*} \]
7.9—Proving a Quadrilateral is a Rhombus, Rectangle, or a Square

**Objective:** Use properties of sides, angles, and diagonals of rhombus, rectangles, and squares.

**Ways to Prove that a Quadrilateral is a Rectangle:**

- Show that it has four right angles
  (Definition of Rect.)

- Show that it is a parallelogram with one right angle.
  \( \square \text{ w/ one rt. } \angle \rightarrow \text{ Rect.} \)

- Show that it is a parallelogram with diagonals that are congruent.
  \( \square \text{ w/ diags } \cong \rightarrow \text{ Rect.} \)
Ways to Prove that a Quadrilateral is a Rhombus:

✓ Show that it has four congruent sides (Definition of Rhombus)

✓ Show that it is a parallelogram with one pair of consecutive sides congruent. (\(\text{\square} \text{ w/ one pair cons. sides } \cong \rightarrow \text{ Rhombus})

✓ Show that it is a parallelogram with diagonals are perpendicular. (\(\text{\square} \text{ w/ diags } \perp \rightarrow \text{ Rhombus})

✓ Show that it is a parallelogram with a diagonal bisects the angles. (\(\text{\square} \text{ w/ diag bisect } \angle s \rightarrow \text{ Rhombus})
Examples:
Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

1. **Given**: $\overline{EF} \cong \overline{FG}, \overline{EG} \perp \overline{FH}$
   **Conclusion**: $EFGH$ is a rhombus.

   The theorems for a rhombus are:
   - w/ one pair cons. sides $\cong \rightarrow$ Rhombus
   - w/ diags $\perp \rightarrow$ Rhombus

   However, to apply either theorem, you must first know that $EFGH$ is a parallelogram, which can’t be proven. Therefore, the conclusion is not valid.
2. Given: $\overline{EB} \cong \overline{BG}, \overline{FB} \cong \overline{BH}, \overline{EG} \cong \overline{FH}, \triangle EBF \cong \triangle EBH$

**Conclusion:** EFGH is a square.

The diagonals bisect each other, so EFGH is a parallelogram.

The diagonals are congruent, so EFGH is a rectangle.

Since $\triangle EBF \cong \triangle EBH$, $\overline{EF} \cong \overline{EH}$.

A pair of consecutive angles are congruent, so EFGH is a rhombus.

Since EFGH is a rectangle and a rhombus, it is a square.
3. Given: $\angle ABC$ is a right angle

**Conclusion:** $ABCD$ is a rectangle.

If one angle of a parallelogram is a right angle, then the parallelogram is a rectangle.

To apply this theorem, you need to know that $ABCD$ is a parallelogram.

Therefore, the conclusion is not valid.
4. Given: \( AB = BC = CD = DA, AC = BD \)

Conclusion: \( ABCD \) is a square

All four sides are congruent, so \( ABCD \) is a rhombus by definition and also a parallelogram.

The diagonals are congruent, so \( ABCD \) is a rectangle.

Since \( ABCD \) is a rhombus and a rectangle, it is also a square.
7.9 Day 2 Warm Up

Use the points $A(-3, 7)$ & $B(5, -3)$ to find the following:

1. Slope  
2. Midpoint  
3. Distance
Objective: Use properties of sides, angles, and diagonals to prove special quadrilaterals.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Sketch</th>
<th>Properties</th>
<th>Coordinate Proofs</th>
<th>Area</th>
</tr>
</thead>
</table>
| Quad. | ![Quad Sketch](image) | • 4 sided polygon  
• Interior \( \angle s \) add to = 360° | | |
| Parallelogram | ![Parallelogram Sketch](image) | • Both pairs opp. sides \( \parallel \)  
• Both pairs opp. sides \( \cong \)  
• Diags. bisect e. o.  
• Both pairs opp. \( \angle s \) \( \cong \)  
• Consec. \( \angle s \) supp. | ➢ Opp. sides same slope  
➢ Opp. sides same distance  
➢ Diags. same midpoint | \( A = b \cdot h \) |
<table>
<thead>
<tr>
<th>Rhombus</th>
<th>4 sides $\cong$</th>
<th>All 4 sides same distance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\square$ w/ diags. $\perp$</td>
<td>$\square$ (diags. same midpt.) AND diags slopes are opp. reciprocals</td>
</tr>
<tr>
<td></td>
<td>$\square$ w/ diags are $\angle$ bisectors</td>
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<tr>
<td></td>
<td>$\square$</td>
<td>$\frac{1}{2} \cdot d_1 \cdot d_2$</td>
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<tr>
<td>Rectangle</td>
<td>4 rt. $\angle$s</td>
<td>Consec. sides slopes are opp. reciprocals</td>
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<td></td>
<td>$\square$ w/ diags $\cong$</td>
<td>$\square$ (diags. same midpt.) AND diags. same distance</td>
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<td></td>
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<td>$A = b \cdot h$</td>
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<tr>
<td>Square</td>
<td>4 sides $\cong$</td>
<td>All sides same distance AND Consec. sides slopes are opp. reciprocals</td>
</tr>
<tr>
<td></td>
<td>4 rt. $\angle$s</td>
<td>$\square$ (diags. same midpt.) AND diags slopes are opp. reciprocals</td>
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<td>$\square$ w/ diags $\perp$</td>
<td>$\square$ (diags. same midpt.) AND diags. same distance</td>
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<td></td>
<td>$\square$ w/ diags $\cong$</td>
<td>$A = s^2$</td>
</tr>
<tr>
<td>Shape</td>
<td>Properties</td>
<td>Area Formula</td>
</tr>
<tr>
<td>------------</td>
<td>------------------------------------------------</td>
<td>-------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Trapezoid</td>
<td>• Exactly one pair of ‖ sides</td>
<td>♦ Only one pair opp. sides have same slope</td>
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<td></td>
<td>♦ Only one pair opp. sides have same slope</td>
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<tr>
<td></td>
<td>♦ Legs are ≅</td>
<td>A = \frac{1}{2}(b_1 + b_2) \cdot h</td>
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<tr>
<td></td>
<td>♦ Diags. are ≅</td>
<td></td>
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<tr>
<td>Isosceles</td>
<td>• One pair opp. sides ‖</td>
<td>♦ Only one pair opp. sides have same slope</td>
</tr>
<tr>
<td>Trapezoid</td>
<td>• Legs are ≅</td>
<td>AND</td>
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<tr>
<td></td>
<td>• Diags. are ≅</td>
<td>♦ Legs same distance or</td>
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<td></td>
<td>♦ Diags same distance</td>
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<tr>
<td>Kite</td>
<td>• 2 pairs consec. sides ≅ (opp. sides not ≅)</td>
<td>♦ 2 pairs consec. sides have the same distance</td>
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<tr>
<td></td>
<td>• Diags are ⊥</td>
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<td></td>
<td>• Only one pair opp. ∠s ≅</td>
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<td></td>
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<td>A = \frac{1}{2}d_1 \cdot d_2</td>
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Use Coordinate Geometry to determine what kind of Parallelogram the coordinates of four vertices make.

- **Diagonals bisect each other**
  - (same midpoint)
  - **Diagonals are perpendicular**
    - (slopes are opposite reciprocals)
  - **Diagonals are congruent**
    - (same distance)
  - (both)
Example: Determine what kind of quadrilateral the four points make.

5. $M (-2, -1)$  $A (1, 3)$  $T (5, 0)$  $H (2, -4)$
6. \( P(-1, 3) \quad Q(-2, 5) \quad R(0, 4) \quad S(1, 2) \)
7. \(L(-1, 1)\) \(M(1, 3)\) \(N(3, 1)\) \(O(1, -3)\)