Chapter 4

Chapter 4 Maintaining Mathematical Proficiency (p. 171)

1. The red figure is a mirror image of the blue figure, so it is a reflection.

2. The blue figure turns to form the red figure, so it is a rotation.

3. The red figure is larger than the blue figure, so it is a dilation.

4. The blue figure slides to form the red figure, so it is a translation.

5. no; Corresponding sides are not proportional.

\[ \frac{5}{7} \neq \frac{12}{14} \]

6. yes; Corresponding angles are congruent, and corresponding side lengths are proportional.

\[
\begin{align*}
\frac{10}{15} &= \frac{2}{3} \\
\frac{8}{12} &= \frac{2}{3} \\
\frac{6}{9} &= \frac{2}{3}
\end{align*}
\]

7. yes; Corresponding angles are congruent and corresponding side lengths are proportional.

\[ \frac{3}{6} = \frac{5}{10} = \frac{1}{2} \]

8. no; Squares have four right angles, so the corresponding angles are always congruent. Because all four sides of a square are congruent, the corresponding sides will always be proportional.

Chapter 4 Mathematical Practices (p. 172)

1. The side lengths are \( AB \approx 4.24, BC \approx 4.12, \) and \( CA \approx 4.12 \) and the angle measurements are \( m\angle B \approx 59.04^\circ, \) \( m\angle A \approx 59.04^\circ, \) and \( m\angle C \approx 61.93^\circ. \)

2. The side lengths are \( AB = 2, BC \approx 5.83, \) and \( CA \approx 5.10 \) and the angle measurements are \( m\angle A \approx 101.31^\circ, \) \( m\angle B \approx 59.04^\circ, \) and \( m\angle C \approx 19.65^\circ. \)

3. The side lengths are \( AB = 4, BC = 3, AD = 3, \) and \( CD = 4 \) and the angle measurements are \( m\angle A = 90^\circ, \) \( m\angle B = 90^\circ, \) \( m\angle C = 90^\circ, \) and \( m\angle D = 90^\circ. \)

4. The side lengths are \( AB = 4, BC \approx 3.16, AD \approx 3.16, \) and \( CD = 4 \) and the angle measurements are \( m\angle A \approx 108.43^\circ, \) \( m\angle B = 71.57^\circ, \) \( m\angle C \approx 108.43^\circ, \) and \( m\angle D \approx 71.57^\circ. \)

5. The side lengths are \( AB \approx 4.24, BC \approx 4.24, AD \approx 4.24, \) and \( CD \approx 4.24 \) and the angle measurements are \( m\angle A = 90^\circ, \) \( m\angle B = 90^\circ, m\angle C = 90^\circ, \) and \( m\angle D = 90^\circ. \)
6. The side lengths are $AB = 4$, $BC \approx 3.16$, $AD = 3$, and $CD \approx 2.24$ and the angle measurements are $m\angle A = 90^\circ$, $m\angle B \approx 18.43^\circ$, $m\angle D \approx 26.57^\circ$, and $m\angle C = 360^\circ - 135^\circ = 225^\circ$.

4.1 Explorations (p. 173)

1. a. and b. Check students' work.
   
   c. The $x$-values of each of the three vertices in the image can be obtained by adding the same amount (positive or negative) to the corresponding $x$-values of the vertices in the original figure. The same is true for the $y$-values.
   
   d. The side lengths and angle measures of the original triangle are equal to the corresponding side lengths and angle measures of the image.

2. a. The rule that determines the translation is if $(x, y)$ are the coordinates of the original point, then $(x + a, y + b)$ are the coordinates of the image of $(x, y)$.
   
   b. $A(0, 3) \rightarrow A'(0 - 4, 3 - 3) = A'(-4, 0)$
   
   $B(4, 5) \rightarrow B'(4 - 4, 5 - 3) = B'(0, 2)$
   
   $C(3, -3) \rightarrow C'(3 - 4, -3 - 3) = C'(-1, -6)$
   
   The coordinates of $\triangle A'B'C'$ are $A'(-4, 0)$, $B'(0, 2)$, and $C'(-1, -6)$.
   
   c. Using the distance formula, determine the lengths of the segments.
   
   $A'B' = \sqrt{(-4 - 0)^2 + (0 - 2)^2}$
   
   $= \sqrt{(-4)^2 + (-2)^2}$
   
   $= \sqrt{16 + 4}$
   
   $= \sqrt{20}$
   
   $= 4.47$
   
   $AB = \sqrt{(0 - 4)^2 + (3 - 5)^2}$
   
   $= \sqrt{(-4)^2 + (-2)^2}$
   
   $= \sqrt{16 + 4}$
   
   $= \sqrt{20}$
   
   $= 4.47$

   $A'C' = \sqrt{(-1 - (-4))^2 + (-6 - 0)^2}$
   
   $= \sqrt{(3)^2 + (-6)^2}$
   
   $= \sqrt{9 + 36}$
   
   $= \sqrt{45}$
   
   $= 6.71$
   
   $AC = \sqrt{(3 - 0)^2 + (-3 - 3)^2}$
   
   $= \sqrt{3^2 + (-6)^2}$
   
   $= \sqrt{9 + 36}$
   
   $= \sqrt{45}$
   
   $= 6.71$
   
   $B'C' = \sqrt{(-1 - 0)^2 + (-6 - 2)^2}$
   
   $= \sqrt{(-1)^2 + (-8)^2}$
   
   $= \sqrt{1 + 64}$
   
   $= \sqrt{65}$
   
   $= 8.06$

   $BC = \sqrt{(3 - 4)^2 + (-3 - 5)^2}$
   
   $= \sqrt{(-1)^2 + (-8)^2}$
   
   $= \sqrt{1 + 64}$
   
   $= \sqrt{65}$
   
   $= 8.06$

   Yes, corresponding sides are equal.

3. a. yes; By the Pythagorean Theorem:
   
   $(AB)^2 + (AC)^2 = (BC)^2$
   
   $(\sqrt{20})^2 + (\sqrt{45})^2 = (\sqrt{65})^2$
   
   $20 + 45 = 65$
   
   $65 = 65$

   b. yes; $ABC$ is a right triangle. Because a translation preserves length and angle measurement, triangle $ABC$ is congruent to triangle $A'B'C'$. Therefore, triangle $A'B'C'$ is a right triangle.

   c. yes; The image is congruent to the original figure, so the corresponding angles will be congruent.

4.5 Summary (p. 177)

5. a. $x = 0$ and $y = 2$
   
   b. $x = 0$ and $y = 3$
   
   c. $x = 0$ and $y = 1$

6. a. The image is congruent to the original figure, so the corresponding angles will be congruent.
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4. To translate a figure, move each vertex of the figure the same number of units left or right, and up or down. Connect the vertices with a straightedge.

5. \( A'(-4 + 3, 0 + 4) = A''(-1, 4) \)
   \( B'(0 + 3, 2 + 4) = B''(3, 6) \)
   \( C'(-1 + 3, -6 + 4) = C''(2, -2) \)
   Translating \( \Delta A'B'C' \) 3 units to the right and 4 units up will yield the following coordinates: \( A''(-1, 4), B''(3, 6), \) and \( C''(2, -2) \). Each vertex of the image is 1 unit left and 1 unit up from the corresponding vertex in the original triangle.

4.1 Monitoring Progress (pp. 174–177)

1. The vector is \( \overrightarrow{BK} \) and the component form is \((-5, 2)\).

2. \( L(2, 2) \rightarrow L'(0, 8) \)
   \( M(5, 3) \rightarrow M'(3, 9) \)
   \( N(9, 1) \rightarrow N'(7, 7) \)

3. The rule to translate \( \Delta A'B'C' \) back to \( \Delta ABC \) is \( (x, y) \rightarrow (x + 4, y - 1) \).

4. \( R(2, 2) \rightarrow R'(3, 4) \)
   \( S(5, 2) \rightarrow S'(6, 4) \)
   \( T(3, 5) \rightarrow T'(4, 7) \)

5. \( T(1, 2) \rightarrow T'(-1, -1) \rightarrow T''(-5, 4) \)
   \( U(4, 6) \rightarrow U'(2, 3) \rightarrow U''(-2, 8) \)

6. \( V(-6, -4) \rightarrow V'(-3, -3) \rightarrow V''(-9, -7) \)
   \( W(-3, 1) \rightarrow W'(0, 2) \rightarrow W''(-6, -2) \)

7. Move the square 1 unit right and 2 units up: 
   \( (x, y) \rightarrow (x + 1, y + 2) \).

4.1 Exercises (pp. 178–180)

Vocabulary and Core Concept Check

1. \( \Delta ABC \) is the preimage, and \( \Delta A'B'C' \) is the image.

2. A translation moves every point of a figure the same distance in the same direction.

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3. The vector is \( \overrightarrow{CD} \) and its component form is \((7, -3)\).

4. The vector is \( \overrightarrow{ST} \) and its component form is \((-2, -4)\).

5. \( D(2, 5) \rightarrow D'(8, 5) \)
   \( E(6, 3) \rightarrow E'(12, 3) \)
   \( F(4, 0) \rightarrow F'(10, 0) \)

6. \( D(2, 5) \rightarrow D'(7, 4) \)
   \( E(6, 3) \rightarrow E'(11, 2) \)
   \( F(4, 0) \rightarrow F'(9, -1) \)

7. \( D(2, 5) \rightarrow D'(-1, -2) \)
   \( E(6, 3) \rightarrow E'(3, -4) \)
   \( F(4, 0) \rightarrow F'(1, -7) \)

8. \( D(2, 5) \rightarrow D'(0, 1) \)
   \( E(6, 3) \rightarrow E'(4, -1) \)
   \( F(4, 0) \rightarrow F'(2, -4) \)

9. The component form of the vector that translates \( P(-3, 6) \) to \( P'(0, 1) \) is \((3, -5)\).
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10. The component form of the vector that translates \( P(-3, 6) \) to \( P'(−4, 8) \) is \((-1, 2)\).

11. \( M(4, 1) \rightarrow M'(−1, 3) \)
\( N(5, -1) \rightarrow N'(0, 1) \)
\( L(1, -1) \rightarrow L'(−4, 1) \)
\[4 + x = -1 \quad 1 + y = 3\]
\[x = -5 \quad y = 2\]
\[5 + x = 0 \quad y + 1 = 1\]
\[x = -5 \quad y = 2\]
\[1 + x = -4 \quad -1 + y = 1\]
\[x = -5 \quad y = 2\]
The rule to translate \( \triangle LMN \) to \( \triangle L'M'N' \) is \((x, y) \rightarrow (x - 5, y + 2)\).

12. \( M(-5, -1) \rightarrow M'(−2, 0) \)
\( N(-2, -4) \rightarrow N'(1, -3) \)
\( L(-6, -4) \rightarrow L'(−3, −3) \)
\[−5 + x = −2 \quad y − 1 = 0\]
\[x = 3 \quad y = 1\]
\[−2 + x = 1 \quad −4 + y = −3\]
\[x = 3 \quad y = 1\]
\[−6 + x = −3 \quad −4 + y = −3\]
\[x = 3 \quad y = 1\]
The rule to translate \( \triangle LMN \) to \( \triangle L'M'N' \) is \((x, y) \rightarrow (x + 3, y + 1)\).

13. Using the translation \((x, y) \rightarrow (x - 8, y + 4)\), the image of \( A(2, 6) \) is \( A'(−6, 10)\).

14. Using the translation \((x, y) \rightarrow (x - 8, y + 4)\), the image of \( B(−1, 5) \) is \( B'(−9, 9)\).

15. \[x - 8 = -3 \quad y + 4 = -10\]
\[\frac{x}{8} + \frac{y}{8} = \frac{-4}{-4}\]
\[x = 5 \quad y = -14\]
The preimage of \( C'(−3, 10) \) is \( C(5, -14)\).

16. \[x - 8 = 4 \quad y + 4 = -3\]
\[x = 12 \quad y = -7\]
The preimage of \( D'(4, -3) \) is \( D(12, -7)\).

17. \[P(-2, 3) \rightarrow P'(2, 9)\]
\[Q(1, 2) \rightarrow Q'(5, 8)\]
\[R(3, -1) \rightarrow R'(7, 5)\]

18. \( P(-2, 3) \rightarrow P'(7, 1)\)
\( Q(1, 2) \rightarrow Q'(10, 0)\)
\( R(3, -1) \rightarrow R'(12, -3)\)

19. \( P(-2, 3) \rightarrow P'(-4, -2)\)
\( Q(1, 2) \rightarrow Q'(-1, -3)\)
\( R(3, -1) \rightarrow R'(1, -6)\)

20. \( P(-2, 3) \rightarrow P'(-3, 6)\)
\( Q(1, 2) \rightarrow Q'(0, 5)\)
\( R(3, -1) \rightarrow R'(2, 2)\)

21. \( X(2, 4) \rightarrow X'(14, 8) \rightarrow X''(9, -1)\)
\( Y(6, 0) \rightarrow Y'(18, 4) \rightarrow Y''(13, -5)\)
\( Z(7, 2) \rightarrow Z'(19, 6) \rightarrow Z''(14, -3)\)

22. \( X(2, 4) \rightarrow X'(-4, 4) \rightarrow X''(-2, 11)\)
\( Y(6, 0) \rightarrow Y'(0, 0) \rightarrow Y''(2, 7)\)
\( Z(7, 2) \rightarrow Z'(1, 2) \rightarrow Z''(3, 9)\)
23. \( \triangle ABC \rightarrow \triangle A'B'C' \)  
\( \triangle A'B'C' \rightarrow \triangle A''B''C'' \)  
\(-4 + x = 1 \quad 2 + y = 3 \quad 1 + x = -4 \quad 3 + y = -2 \)  
\( x = 5 \quad y = 1 \quad x = -5 \quad y = 5 \)  
\(-1 + x = 4 \quad 1 + y = 2 \quad 4 + x = -1 \quad 2 + y = -3 \)  
\( x = 5 \quad y = 1 \quad x = -5 \quad y = 5 \)  
\(-4 + x = 1 \quad 1 + y = 2 \quad 1 + x = -4 \quad 2 + y = -3 \)  
\( x = 5 \quad y = 1 \quad x = -5 \quad y = 5 \)  
The translation from \( \triangle ABC \) to \( \triangle A'B'C' \) is 
\( (x, y) \rightarrow (x + 5, y + 1) \) and the translation from \( \triangle A'B'C' \) to \( \triangle A''B''C'' \) is \( (x, y) \rightarrow (x - 5, y - 5) \).

24. \( \text{DEFG} \rightarrow \text{D'E'F'G'} \)  
\( \text{D'E'F'G'} \rightarrow \text{D''E''F''G''} \)  
\(-5 + x = 1 \quad 3 + y = -1 \quad 1 + x = -5 \quad -1 + y = -1 \)  
\( x = 6 \quad y = -4 \quad x = -6 \quad y = 0 \)  
\(-3 + x = 3 \quad 3 + y = -1 \quad 3 + x = -3 \quad -1 + y = -1 \)  
\( x = 6 \quad y = -4 \quad x = -6 \quad y = 0 \)  
\(-1 + x = 5 \quad 1 + y = -3 \quad 5 + x = -1 \quad -3 + y = -3 \)  
\( x = 6 \quad y = -4 \quad x = -6 \quad y = 0 \)  
\(-5 + x = 1 \quad 1 + y = -3 \quad 1 + x = -5 \quad -3 + y = -3 \)  
\( x = 6 \quad y = -4 \quad x = -6 \quad y = 0 \)  
The translation from \( \text{DEFG} \) to \( \text{D'E'F'G'} \) is 
\( (x, y) \rightarrow (x + 6, y - 4) \) and the translation from \( \text{D'E'F'G'} \) to \( \text{D''E''F''G''} \) is \( (x, y) \rightarrow (x - 6, y) \).

25. \( \text{E'F'G'H'} \) should have been translated left and down. 
\( 
\begin{align*}
E(3, 2) & \rightarrow E'(3 - 1, 2 - 2) \rightarrow E'(2, 0) \\
F(7, 3) & \rightarrow F'(7 - 1, 3 - 2) \rightarrow F'(6, 1) \\
G(7, 0) & \rightarrow G'(7 - 1, 0 - 2) \rightarrow G'(6, -2) \\
H(4, 1) & \rightarrow H'(4 - 1, 1 - 2) \rightarrow H'(3, -1)
\end{align*}
\)

26. 1st translation: \( (x, y) \rightarrow (x + 2, y - 1) \)  
2nd translation: \( (x, y) \rightarrow (x + 1, y - 2) \)  
Composite translation: \( (x, y) \rightarrow (x + 3, y - 3) \)

27. a. The amoeba moves right 5 squares and down 4 squares.  
\[10^2 + (-8)^2 = c^2\]  
\[100 + 64 = c^2\]  
\[\sqrt{164} = c\]  
\[c = 12.81\]

The amoeba travels about 12.81 millimeters.

b. \( 12.81 \text{ mm} = 0.52 \text{ mm/sec} \)

The amoeba moves about 0.52 millimeter per second.

28. a. \( A: (x, y) \rightarrow (x + n, y + t)\)  
\( B: (x, y) \rightarrow (x + s, y + m)\)

Translation A followed by translation B is  
\( (x, y) \rightarrow (x + n + s, y + t + m)\).

b. \( A: (x, y) \rightarrow (x + n, y + t)\)  
\( B: (x, y) \rightarrow (x + s, y + m)\)

Translation B followed by translation A is  
\( (x, y) \rightarrow (x + s + n, y + m + t)\).

c. no; Because addition is commutative, \( s + n \) is the same as \( n + s \), and \( m + t \) is the same as \( t + m \). So, each image will end up in the same place.

29. The blue figure is congruent to the red figure. \( r = 100 \) because corresponding angles are congruent.  
\( 162 = 3w \rightarrow w = 54 \) because corresponding angles are congruent. \( 2t = 10 \rightarrow t = 5 \) because corresponding sides are congruent. \( s = 8 \) because corresponding sides are congruent.

30. The blue triangle is congruent to the red triangle.  
\( 90^\circ - 55^\circ = 55^\circ \rightarrow a = 35 \)  
\( b + 6 = 20 \)
\[ b = 14 \]
\[ 4c - 6 = 14 \]
\[ 4c = 20 \]
\[ c = 5 \]

31. \( \text{D}(-1, 2) \) and \( \text{D'}(-2, -2) \)
\[ -1 + x = -2 \quad 2 + y = -2 \]
\[ x = -1 \quad y = -4 \]

So, the component form of the translation is \((-1, -4)\).  
\( \text{E'}(-2 + (-1), 0 + (-4)) \rightarrow (-3, -4) \)  
\( \text{F'}(-1 + (-1), -1 + (-4)) \rightarrow (-2, -5) \)  
\( \text{G'}(1 + (-1), 3 + (-4)) \rightarrow (0, -1) \)

32. The two figures that are translations of each other are figures 5 and 7. The translation of figure 5 to figure 7 is 4 units right and 8 units up, \( (x, y) \rightarrow (x + 4, y + 8) \).

33. \( (x_2, y_2) \rightarrow (x_1 + m, y_1 + n) \)
\[ x_2 = x_1 + m \quad y_2 = y_1 + n \]
\[ x_1 = x_2 - m \quad y_1 = y_2 - n \]

The rule to map \( \text{PQ} \) to \( \text{P'}Q' \) is \( (x, y) \rightarrow (x - m, y - n) \).
34. a. \( Q(2, -3) \rightarrow Q'(2 - 3, -3 - 3) = Q'(-1, -6) \)
    \( R(2, 4) \rightarrow R'(2 - 3, 4 - 3) = R'(-1, 1) \)
    \( S(5, 4) \rightarrow S'(5 - 3, 4 - 3) = S'(2, 1) \)
    \( T(5, -3) \rightarrow T'(5 - 3, -3 - 3) = T'(2, -6) \)
    \( QT = |5 - 2| = |3| = 3 \)
    \( TS = |4 - (-3)| = |7| = 7 \)
    Area of \( Q'R'S'T' \) = 3 \( \cdot \) 7 = 21 square units
    \( QT' = |-1 - 2| = |-3| = 3 \)
    \( T'S' = |-6 - 1| = |-7| = 7 \)
    Area of \( Q'R'S'T' \) = 3 \( \cdot \) 7 = 21 square units
    The area of rectangle \( QSTR' \) is 21 square units and the area of rectangle \( Q'R'S'T' \) is 21 square units.

b. The areas of the translated figures are equal. The preimage and the image of a translation are congruent, so their areas are congruent.

35. Given the translation \( \triangle ABC \rightarrow \triangle A'B'C' \) and \( \triangle A'B'C' \rightarrow \triangle A''B''C'' \), prove that the composition translation \( \triangle ABC \rightarrow \triangle A''B''C'' \) is a rigid motion. A translation is defined as a rigid motion that preserves length and angle measures. Therefore, the translation of \( \triangle ABC \rightarrow \triangle A'B'C' \) will yield \( \triangle ABC \equiv \triangle A'B'C' \), where the corresponding sides and angles are congruent. \( \overline{AB} \equiv \overline{A'B'} \), \( \overline{AC} \equiv \overline{A'C'} \), \( \overline{BC} \equiv \overline{B'C'} \), \( \angle A \equiv \angle A' \), \( \angle B \equiv \angle B' \), and \( \angle C \equiv \angle C' \). Likewise, the translation \( \triangle A'B'C' \rightarrow \triangle A''B''C'' \) will yield \( \overline{A'B'} \equiv \overline{A''B''} \), \( \overline{A'C'} \equiv \overline{A''C''} \), \( \overline{B'C'} \equiv \overline{B''C''} \), \( \angle A' \equiv \angle A'' \), \( \angle B' \equiv \angle B'' \), and \( \angle C' \equiv \angle C'' \). By the transitive property, \( \overline{AB} \equiv \overline{A'B'} \), \( \overline{AC} \equiv \overline{A'C'} \), \( \overline{BC} \equiv \overline{B'C'} \), \( \angle A \equiv \angle A' \), \( \angle B \equiv \angle B' \), and \( \angle C \equiv \angle C' \). The composition of two translations is itself a translation. So, by the definition of rigid motion, the composition of two or more (\( \triangle ABC \rightarrow \triangle A''B''C'' \)) rigid motions is a rigid motion.

36. a. 

Because \( \overline{AB} \parallel \overline{CD} \) and translations map lines to parallel lines, a translation along \( \overline{BD} \) maps \( \overline{AB} \) to \( \overline{CD} \).

Because translations are rigid motions, angle measures are preserved, which means the angles formed by \( \overline{AB} \) and \( \overline{BD} \) are congruent to the corresponding angles formed by \( \overline{CD} \) and \( \overline{BD} \).

b. Cross two coplanar lines with a transversal.

Because \( \angle ABD \equiv \angle CDE \), a translation along \( \overline{BD} \) maps \( \triangle ABD \) onto \( \triangle CDE \). Because translations map lines to parallel lines, \( \overline{AB} \parallel \overline{CD} \).

37. Draw a rectangle using the vertices \( A(x_1, y_1), B(x_2, y_2), C(x_3, y_3), \) and \( D(x_4, y_4) \). Add to each \( x \)-coordinate a value of \( h \) and to each \( y \)-coordinate a value of \( k \): \( A'(x_1 + h, y_1 + k), B'(x_2 + h, y_2 + k), C'(x_3 + h, y_3 + k), \) and \( D'(x_4 + h, y_4 + k) \). Then connect \( A \rightarrow A', B ightarrow B', C ightarrow C', \) and \( D ightarrow D' \). Finally, make the hidden lines dashed.

38. \( A(-1, w) \rightarrow A'(-1 + 4, w + 1) = A'(2x + 1, 4) \)
    \(-1 + 4 = 2x + 1 \) \( \rightarrow \) \( w + 1 = 4 \)
    \(-1 = 2x + 1 \) \( \rightarrow \) \( w = 3 \)
    \( 3 = 2x + 1 \) \( \rightarrow \) \( 2 = 2x \)
    \( x = 1 \)
    \( B(8y - 1, 1) \rightarrow B'(8y - 1 + 4, 1 + 1) = B'(3, 3z) \)
    \( 8y - 1 + 4 = 3 \) \( \rightarrow \) \( 1 + 1 = 3z \)
    \( 8y = 3 \) \( \rightarrow \) \( z = \frac{3}{4} \)
    \( 8y = 0 \) \( \rightarrow \) \( z = \frac{3}{4} \)
    \( y = 0 \)

39. yes; According to the definition of translation, the segments connecting corresponding vertices will be congruent and parallel. Also, because a translation is a rigid motion, \( \overline{GH} \equiv \overline{GH'} \). So, the resulting figure is a parallelogram.

40. Sample answer:
41. no; Because the value of y changes, you are not adding the same amount to each x-value.

42. STATEMENTS

1. MN is perpendicular to line ℓ.
2. M′N′ is the translation of MN 2 units to the left.
3. If M(x₁, y₁) and N(x₂, y₂), then M′(x₁ − 2, y₁) and N′(x₂ − 2, y₂).
4. \( m_{\overline{MN}} = \frac{y₂ - y₁}{x₂ - x₁} \) and

\[ m_{\overline{M′N′}} = \frac{y₂ - y₁}{(x₂ - 2) - (x₁ - 2)} = \frac{y₂ - y₁}{x₂ - x₁} \]

5. \( m_{\overline{MN}} = m_{\overline{M′N′}} \)
6. \( \overline{MN} \parallel \overline{M′N′} \)
7. \( \overline{M′N′} \perp ℓ \)

REASONS

1. Given
2. Given
3. Definition of translation
4. Definition of slope
5. Transitive Property of Equality
6. Slopes of Parallel Lines (Thm. 3.13)
7. Perpendicular Transversal Theorem (Thm. 3.11)

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43. yes; The figure can be folded in half in several ways so that one side matches the other.

44. no

46. yes; The figure can be folded in half so that one side matches the other.

47. \(-(-x) = x\)

48. \(-(x + 3) = -x - 3\)

49. \(x - 12 + 5x = 6x - 12\)

50. \(x + 2x - 4 = 3x - 4\)

4.2 Explorations (p. 181)

1. a. Check students’ work.
   b. Check students’ work.
   c. Sample answer:

   \[
   \begin{array}{c}
   A \quad B \\
   C \quad B' \\
   A' \\
   \end{array}
   \]

   \[
   \begin{array}{c}
   C \quad B' \\
   B' \\
   A' \quad C \\
   \end{array}
   \]

   2. a. Check students’ work.

   b. Each vertex of \(\triangle A'B'C'\) has the same y-value as its corresponding vertex of \(\triangle ABC\). The x-value of each vertex of \(\triangle A'B'C'\) is the opposite of the x-value of its corresponding vertex of \(\triangle ABC\).

   c. The corresponding side lengths and corresponding angle measures are congruent.

   d. Sample answer:

   \[
   \begin{array}{c}
   A \quad B \\
   C \quad B' \\
   A' \quad C \\
   \end{array}
   \]

   Each vertex of \(\triangle A'B'C'\) has the same x-value as its corresponding vertex of \(\triangle ABC\). The y-value of each vertex of \(\triangle A'B'C'\) is the opposite of the x-value of its corresponding vertex of \(\triangle ABC\); The corresponding sides and corresponding angles are congruent.

   3. If a figure is reflected in the y-axis, then each pair of corresponding vertices will have the same y-value and opposite x-values. If a figure is reflected in the x-axis, then each pair of corresponding vertices will have the same x-value and opposite y-values.
4.2 Monitoring Progress (pp. 182–185)

1. Point A is 3 units to the left of $x = 4$, so $A'$ is 3 units to the right of $x = 4$ at $(7, 3)$. $B'$ is 1 unit to the left of $x = 4$ at $(3, 2)$, and $C'$ is 2 units to the right of $x = 4$ at $(6, 1)$.

2. Point A is 4 units to the right of $x = -3$, so $A'$ is 4 units to the left of $x = -3$ at $(-7, 3)$. $B'$ is 8 units to the left of the $x$-axis at $(-11, 2)$, and $C'$ is 5 units to the left of $x = -3$ at $(-8, 1)$.

3. Point A is 2 units above $y = 2$, so $A'$ is 2 units below $y = 2$ at $(1, 1)$. Because point B is on the line $y = 2$, $B = B'$. $C'$ is 1 unit above $y = 2$ at $(2, 3)$.

4. Point A is 4 units above $y = -1$, so $A'$ is 4 units below $y = -1$ at $(1, -5)$. $B'$ is 3 units below $y = -1$ at $(5, -4)$, and $C'$ is 2 units below $y = -1$ at $(2, -3)$.

5. Reflect $\triangle JKL$ in the $x$-axis:
   $J(1, 3) \rightarrow J'(1, -3)$, $K(4, 4) \rightarrow K'(4, -4)$, $L(3, 1) \rightarrow L'(3, -1)$

6. Reflect $\triangle JKL$ in the $y$-axis:
   $J(1, 3) \rightarrow J'(-1, 3)$, $K(4, 4) \rightarrow K'(-4, 4)$, $L(3, 1) \rightarrow L'(-3, 1)$

7. Image of $\triangle JKL$ after the reflection in the line $y = x$:
   $J(1, 3) \rightarrow J'(3, 1)$, $K(4, 4) \rightarrow K'(4, 4)$, $L(3, 1) \rightarrow L'(1, 3)$

8. Image of $\triangle JKL$ after the reflection in the line $y = -x$:
   $J(1, 3) \rightarrow J'(-3, -1)$, $K(4, 4) \rightarrow K'(-4, -4)$, $L(3, 1) \rightarrow L'(-1, -3)$

9. $F(-1, 2), F'(-2, 1)$
   Slope of $FF' = \frac{1 - 2}{-2 - (-1)} = -1 = \frac{-1}{1}$
   For two lines to be perpendicular, the slopes must be negative reciprocals of each other. Because $y = -x$ has a slope of $-1$ and the slope of $FF' = 1$, $FF'$ is perpendicular to $y = -x$.

10. Begin by graphing $\triangle ABC$ and translating 4 units down: $A(3, 2) \rightarrow A'(3, -2)$, $B(6, 3) \rightarrow B'(6, -1)$, and $C(7, 1) \rightarrow C'(7, -3)$. Then reflect $\triangle A'B'C'$ in the $y$-axis:
    $A'(3, -2) \rightarrow A''(-3, -2)$, $B'(6, -1) \rightarrow B''(-6, -1)$, and $C'(7, -3) \rightarrow C''(-7, -3)$. 
Chapter 4

11. The glide reflection from △A′B′C′ to △ABC consists of translating 12 units right and reflecting in the x-axis.

12. There are two lines of symmetry.

13. There are five lines of symmetry.

14. There is one line of symmetry.

15. Sample answer:

16. Reflect A in line m to obtain A′. Then draw A′B. Label the intersection of A′B and m as C. Because A′B is the shortest distance between B and A′ and AC = A′C, park at point C.

4.2 Exercises (pp. 186–188)

Vocabulary and Core Concept Check

1. A glide reflection is a combination of a translation and a reflection.

2. The second transformation does not belong because it is a translation and the other three are reflections.

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3. Reflection in the y-axis

4. Neither

5. Neither

6. Reflection in the x-axis

7. Reflect △JKL in the x-axis: J(2, -4) → J′(2, 4), K(3, 7) → K′(3, -7), L(6, -1) → L′(6, 1)

8. Reflect △JKL in the y-axis: J(5, 3) → J′(−5, 3), K(1, -2) → K′(−1, -2), L(−3, 4) → L′(3, 4)

9. Reflect △JKL in x = -1: J(2, -1) → J′(−4, −1), K(4, -5) → K′(−6, −5), L(3, 1) → L′(−5, 1)

10. Reflect △JKL in x = 2: J(1, −1) → J′(3, −1), K(3, 0) → K′(1, 0), L(0, −4) → L′(4, −4)

11. Reflect △JKL in y = 1: J(2, 4) → J′(2, −2), K(−4, −2) → K′(−4, 4), L(−1, 0) → L′(−1, 2)
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12. Reflect $\triangle JKL$ in $y = -3$: $J(3, -5) \rightarrow J'(3, -1)$, $K(4, -1) \rightarrow K'(4, -5)$, $L(0, -3) \rightarrow L'(0, -3)$

13. Reflecting $\triangle ABC$ in $y = x$, the original point is $(a, b)$ and the image point is $(b, a)$.

14. Reflecting $\text{ABCD}$ in $y = x$, the original point is $(a, b)$ and the image point is $(b, a)$.

15. Reflecting $\text{ABCD}$ in $y = -x$, the original point is $(a, b)$ and the image point is $(-b, -a)$.

16. Reflecting $\triangle ABC$ in $y = -x$, the original point is $(a, b)$ and the image point is $(-b, -a)$.

17. Translation $(x, y) \rightarrow (x, y - 1)$: $T(6, 4) \rightarrow T'(6, 3)$, $S(7, 3) \rightarrow S'(7, 2)$, $R(4, 1) \rightarrow R'(4, 0)$

Reflection in the $y$-axis: $T'(6, 3) \rightarrow T''(-6, 3)$, $S'(7, 2) \rightarrow S''(-7, 2)$, $R'(4, 0) \rightarrow R''(-4, 0)$

18. Translation $(x, y) \rightarrow (x - 3, y)$: $T(6, 4) \rightarrow T'(3, 4)$, $S(7, 3) \rightarrow S'(4, 3)$, $R(4, 1) \rightarrow R'(1, 1)$

Reflection in the line $y = 1$: $T'(3, 4) \rightarrow T''(3, -6)$, $S'(4, 3) \rightarrow S''(4, -5)$, $R'(1, 1) \rightarrow R''(1, -3)$

19. Translation $(x, y) \rightarrow (x, y + 4)$: $T(6, 4) \rightarrow T'(6, 8)$, $S(7, 3) \rightarrow S'(7, 7)$, $R(4, 1) \rightarrow R'(4, 5)$

Reflection in the line $x = 3$: $T'(6, 8) \rightarrow T''(0, 8)$, $S'(7, 7) \rightarrow S''(-1, 7)$, $R'(4, 5) \rightarrow R''(2, 5)$

20. Translation $(x, y) \rightarrow (x + 2, y + 2)$: $T(6, 4) \rightarrow T'(8, 6)$, $S(7, 3) \rightarrow S'(9, 5)$, $R(4, 1) \rightarrow R'(6, 3)$

Reflection in the line $y = x$: $T'(8, 6) \rightarrow T''(6, 8)$, $S'(9, 5) \rightarrow S''(5, 9)$, $R'(6, 3) \rightarrow R''(3, 6)$
21. There is one line of symmetry.

22. There are four lines of symmetry.

23. There are no lines of symmetry.

24. There are five lines of symmetry.

25. a. There are no lines of symmetry.
   b. The line of symmetry can be drawn down the middle of the O.
   c. The lines of symmetry can be drawn horizontally centered through the O and X.
   d. There are no lines of symmetry.

26. The line of reflection is incorrect. After translating \((x + 2, y + 3)\), the reflection is in the y-axis. The line of reflection must be parallel to the direction of the translation.

27. Reflect \(H\) in line \(n\) to obtain \(H'\). Connect \(H'\) to \(J\) to draw \(JH'\). Label the intersection of \(JH'\) and \(n\) as \(K\). Because \(JH'\) is the shortest distance between \(J\) and \(H'\) and \(HK = H'K\), park at point \(K\).

28. The translation is \((x, y) \rightarrow (x + 3, y + 3)\).

29. Reflect \(A\) in the x-axis so the line \(y = 0\) and \(A'B'\) intersect. \(A(1, 4), B(6, 1), A'(1, -4)\)
   
   Using the slope-intercept form of the equation of a line, \(y = mx + b\), determine the equation of the line that contains \(A'B'\).
   
   Slope of \(A'B'\): \(m = \frac{1 - (-4)}{6 - 1} = \frac{5}{5} = 1\)
   
   To determine the y-intercept, substitute the slope, 1, and a point on the line, \((1, -4)\).
   
   \(y = mx + b\)
   
   
   \(-4 = 1 + b\)
   
   \(b = -5\)
   
   So, the equation is \(y = x - 5\). To determine the intersection of \(A'B'\) and \(y = 0\), set the equations equal to each other and solve for \(x\).
   
   \(0 = x - 5\)
   
   \(x = 5\)
   
   So, the point that minimizes \(AC + BC\) is \(C(5, 0)\).

30. The line segment connecting \(A(4, -5)\) to \(B(12, 3)\) crosses the x-axis. Using the slope-intercept form of the equation of a line, \(y = mx + b\), determine the equation of the line that contains \(AB\).
   
   Slope of \(AB\): \(m = \frac{3 - (-5)}{12 - 4} = \frac{8}{8} = 1\)
   
   To determine the y-intercept, substitute the slope, 1, and a point on the line, \((4, -5)\).
   
   \(y = mx + b\)
   
   
   
   \(-5 = 1(4) + b\)
   
   \(b = -9\)
   
   So, the equation is \(y = x - 9\). To determine the intersection of \(AB\) and \(y = 0\), set the equations equal to each other and solve for \(x\).
   
   \(0 = x - 9\)
   
   \(x = 9\)
   
   So, the point that minimizes \(AC + BC\) is \(C(9, 0)\).
31. Reflect \( A \) in the \( x \)-axis so the line \( y = 0 \) and \( A'B' \) intersect. 
\( A(-8, 4), B(-1, 3), A'(8, -4) \)
Using the slope-intercept form of the equation of a line, 
\( y = mx + b \), determine the equation of the line that contains \( A'B' \).
Slope of \( A'B' \): 
\[
\frac{-4 - 3}{8 - (-1)} = \frac{-7}{9} \]
To determine the \( y \)-intercept, substitute the slope, \( 1 \), and a point on the line, \((-1, 3)\).
\[
y = mx + b \\
3 = -1 + b \\
b = 4
\]
So, the equation is \( y = x + 4 \). To determine the intersection of \( A'B' \) and \( y = 0 \), set the equations equal to each other and solve for \( x \).
\[
0 = x + 4 \\
x = -4
\]
So, the point that minimizes \( AC + BC \) is \((-4, 0)\).

32. The line segment connecting \( A(-1, 7) \) to \( B(5, -4) \) crosses the \( x \)-axis. Using the slope-intercept form of the equation of a line, \( y = mx + b \), determine the equation of the line that contains \( AB \).
Slope of \( AB \): 
\[
\frac{-4 - 7}{5 - (-1)} = \frac{-11}{6}
\]
To determine the \( y \)-intercept, substitute the slope, \( \frac{-11}{6} \), and a point on the line, \((-1, 7)\).
\[
y = \frac{-11}{6}x + b \\
7 = \frac{-11}{6}(-1) + b \\
b = 4
\]
So, the equation is \( y = \frac{-11}{6}x + 4 \). To determine the intersection of \( AB \) and \( y = 0 \) set the equations equal to each other and solve for \( x \).
\[
0 = \frac{-11}{6}x + 4 \\
0 = -11x + 24 \\
-31 = -11x \\
x = \frac{31}{-11} \]
So, the point that minimizes \( AC + BC \) is \( \left( \frac{31}{11}, 0 \right) \).

33. The \( y \)-intercept of the preimage is 3 units above the line \( y = -1 \). So, the \( y \)-intercept of the image is 3 units below the line \( y = -1 \), or \((0, -4)\). The point \((-1, -1)\) is on the line of the preimage and is on the line of reflection, so it is also on the line of the image. So, the slope of the line of the image is
\[
m = \frac{-1 - (-4)}{-1 - 0} = -3
\]
So, the equation of the image of the reflection of \( y = 3x + 2 \) is \( y = -3x - 4 \).

34. a. Figure 2 is a reflection of Figure A in the line \( x = a \), where \( a = 3.5 \). A reflection in a line \( x = a \) will keep the \( y \)-values the same, and the \( x \)-values will be the same distance from the line of reflection, but on opposite sides: 
\((1, 2) \rightarrow (6, 2), (3, 4) \rightarrow (4, 4), (2, 6) \rightarrow (5, 6)\).

b. Figure 4 is a reflection of Figure A in the line \( y = b \), where \( b = 3 \). A reflection in a line \( y = b \) will keep the \( x \)-values the same, and the \( y \)-values will be the same distance from the line of reflection, but on opposite sides: 
\((1, 2) \rightarrow (1, 4), (3, 4) \rightarrow (3, 2), (2, 6) \rightarrow (2, 0)\).

c. Figure 1 is a reflection of Figure A in the line \( y = x \). A reflection in a line \( y = x \) will reverse the \( x \) and \( y \)-coordinates: 
\((1, 2) \rightarrow (2, 1), (3, 4) \rightarrow (4, 3), (2, 6) \rightarrow (6, 2)\).

d. Figure 3 represents a glide reflection of Figure A. The translation is \((x, y) \rightarrow (x, y - 2)\), which results in the values: 
\((1, 2) \rightarrow (1, 0), (3, 4) \rightarrow (3, 2), (2, 6) \rightarrow (2, 4)\). The reflection is \(x = 3.5\), which results in the values: 
\((1, 0) \rightarrow (6, 0), (3, 2) \rightarrow (4, 2), (2, 4) \rightarrow (5, 4)\).

35. 

36. Line up the reflective device on line \( m \) to verify that \( \triangle ABC \) reflects onto \( \triangle A'B'C' \) and that \( \triangle ABC \equiv \triangle A'B'C' \).

37. Using a reflective device or dynamic geometry software, plot the points \( M, N, \) and \( Q \). Then construct the line \( y = -2x \), and reflect in the line to obtain the points: 
\[
M(0, 3) \rightarrow M'(2.4, 1.8), N(-1, -1) \rightarrow N'(1.4, 0.2), Q(-5, 0) \rightarrow Q'(3, 4)\).

38. no; Sample answer: A counterexample is as follows.
translation: \((x, y) \rightarrow (x, y - 2)\)
reflection: in the \( x \)-axis
This composition is not commutative. However, all glide reflections are commutative because the line of reflection is parallel to the direction of the translation.
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4.3 Explorations (p. 189)

1. a. Check students’ work.
   
   b. Check students’ work.
   
   c. The x-value of each vertex of \( \triangle A'B'C' \) is the opposite of the y-value of its corresponding vertex in \( \triangle ABC \). The y-value of each vertex of \( \triangle A'B'C' \) is equal to the x-value of its corresponding vertex in \( \triangle ABC \).
   
   d. The corresponding lengths and the corresponding angles are congruent. For example, \( AB = A'B' \) and \( m\angle A = m\angle A' \).

2. a. The rule to rotate \((x, y)\) \(90^\circ\) about the origin is \((x, y) \to (-y, x)\).

   b. The coordinates of the vertices of the image of \( \triangle A'B'C' \) are \(A(0, 3) \to A'(-3, 0), B(4, 5) \to B'(-5, 4), C(3, -3) \to C'(3, 3)\).

   c. Use the Distance Formula to find the side lengths of both triangles.

   \[
   AB = \sqrt{(0 - 4)^2 + (3 - 5)^2} = \sqrt{16 + 4} = \sqrt{20}
   \]

   \[
   A'B' = \sqrt{(-3 - (-5))^2 + (0 - 4)^2} = \sqrt{4 + 16} = \sqrt{20}
   \]

   \[
   BC = \sqrt{(4 - 3)^2 + [5 - (-3)]^2} = \sqrt{1 + 64} = \sqrt{65}
   \]

3. a. The rule to rotate \((x, y)\) \(180^\circ\) about the origin is \((x, y) \to (-x, -y)\). When a point is rotated \(180^\circ\), the x-value and y-value of the image are the opposite of the x-value and y-value of the original point.

   b. The coordinates of \( \triangle A'B'C' \) after a rotation of \(180^\circ\) are \(A(0, 3) \to A'(0, -3), B(4, 5) \to B'(-4, -5), C(3, -3) \to C'(-3, 3)\).

4. Sample answer: Put your pencil on the origin and rotate the graph the given number of degrees. Record the coordinates of the image in this orientation. Then return the coordinate plane to its original orientation, and draw the image using the coordinates you recorded.

5. The coordinates of the vertices of \( \triangle A'B'C' \) are \(A'(0, -3), B'(-4, -5), \) and \(C'(-3, 3)\). The coordinates of the image \( \triangle A''B''C'' \) are the same as those of \( \triangle ABC \):

   \[
   A''(0, 3) \to A'(0, -3) = A(0, 3), B'(4, 5) \to B''(4, 5) = B(4, 5), \]

   and \(C'(-3, 3) \to C''(3, -3) = C(3, -3)\).
4.3 Monitoring Progress (pp. 190–193)

1. Draw a segment from $P$ to $E$. Draw a ray to form a 50° angle with $PE$. Place $E'$ so that $PE' = PE$. Repeat the steps for points $D'$ and $F'$.

2. The vertices of the image $\triangle J'K'L'$ are $J'(0, 3)$, $L'(0, 6)$, and $K'(-3, 4)$.

3. After reflecting $RS$ in the x-axis, the endpoints of $R'S'$ are $R'(1, 3)$ and $S'(2, 6)$. Rotating $R'S'$ 180° about the origin produces $R''(-1, -3)$ and $S''(-2, -6)$.

5. Applying the translation $(x - 2, y - 1)$ to $AB$, $A'B''$ has the endpoints $A'(-6, 3)$ and $B'(-3, 6)$. Rotating $A'B''$ 90° about the origin, $A''B''$ has endpoints $A''(-3, -6)$ and $B''(-6, -3)$.

6. Rotating $\triangle TUV$ about the origin 90°, $\triangle T'U'V'$ has the vertices $T'(-1, -2)$, $U'(-3, -5)$, and $V'(-6, -3)$. Reflecting $\triangle T'U'V'$ in the x-axis, $\triangle T''U''V''$ has vertices $T''(-1, 2)$, $U''(-3, 5)$, and $V''(-6, 3)$.

7. The rhombus has rotational symmetry. The center is the intersection of the diagonals. A 180° rotation about the center maps the rhombus onto itself.

8. The octagon has rotational symmetry. The center is the intersection of the diagonals. A 90° or 180° rotation about the center maps the octagon onto itself.

9. The right triangle has no rotational symmetry.

4.3 Exercises (pp. 194–196)

Vocabulary and Core Concept Check

1. When a point $(a, b)$ is rotated counterclockwise about the origin, $(a, b) \to (b, -a)$ is the result of a 270° rotation.

2. The rotations about the origin, 90° counterclockwise, 270° clockwise, and a 90° rotation to the left all yield the same result: $A'(-2, 1)$, $B'(-4, 2)$, $C'(-2, 4)$. So, “What are the coordinates of the vertices of the image after a 270° clockwise rotation about the origin?” is different. A rotation of 270° counterclockwise about the origin yields $A'(2, -1)$, $B'(4, -2)$, and $C'(2, -4)$.
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3. Draw \( \triangle ABC \) and point \( P \). Draw a segment from \( P \) to \( A \). Using a protractor, draw a ray to form a 30° angle with \( PA \). Place \( A' \) so that \( PA = PA' \). Repeat the steps by drawing a segment from \( P \) to \( B \) and drawing a ray to form a 30° angle with \( PB \). Place \( B' \) so that \( PB = PB' \). Repeat the steps by drawing a segment from \( P \) to \( C \) and drawing a ray to form a 30° angle with \( PC \). Place \( C' \) so that \( PC = PC' \). Connect the vertices \( A', B', \) and \( C' \).

4. Draw quadrilateral \( DEFG \) and point \( P \). Draw a segment from \( P \) to \( E \). Using a protractor, draw a ray to form an 80° angle with \( PE \). Place \( E' \) so that \( PE = PE' \). Repeat this process for points \( D, F, \) and \( G \) to complete the rotation of the quadrilateral \( DEFG \) and connect \( D', E', F', \) and \( G' \).

5. Draw quadrilateral \( FGJP \). Lay the protractor on the side of \( PJ \) and draw a ray to form a 150° angle with \( PJ \). Place \( J' \) so that \( PJ = PJ' \). Repeat this process for points \( F \) and \( G \) to complete the rotation of the quadrilateral \( FGJP \). \( P \) will stay in the same position. Connect \( F', G', J', \) and \( P \).

6. Draw \( \triangle PRQ \). Lay the protractor on the side of \( PR \) and draw a ray to form a 130° angle. Place \( R' \) so that \( PR = PR' \). Repeat the steps by drawing a segment from \( P \) to \( Q \) and drawing a ray to form a 130° angle with \( PQ = PQ' \). \( P \) will stay in the same position. Connect \( P', Q', \) and \( P \).

7. Use the coordinate rule for a 90° rotation around the origin, \((a, b) \rightarrow (-b, a)\).
\[
\begin{align*}
A(-3, 2) & \rightarrow A'(-2, -3) \\
B(2, 4) & \rightarrow B'(-4, 2) \\
C(3, 1) & \rightarrow C'(-1, 3)
\end{align*}
\]

8. Use the coordinate rule for a 180° rotation around the origin, \((a, b) \rightarrow (-a, -b)\).
\[
\begin{align*}
D(-3, -1) & \rightarrow D'(3, 1) \\
E(-1, 2) & \rightarrow E'(1, -2) \\
F(4, -2) & \rightarrow F'(-4, 2)
\end{align*}
\]

9. Use the coordinate rule for a 180° rotation around the origin, \((a, b) \rightarrow (-a, -b)\).
\[
\begin{align*}
J(1, 4) & \rightarrow J'(-1, -4) \\
K(5, 5) & \rightarrow K'(-5, -5) \\
L(7, 2) & \rightarrow L'(-7, -2) \\
M(2, 2) & \rightarrow M'(-2, -2)
\end{align*}
\]

10. Use the coordinate rule for a 270° rotation around the origin, \((a, b) \rightarrow (b, -a)\).
\[
\begin{align*}
Q(-6, -3) & \rightarrow Q'(3, 6) \\
R(-5, 0) & \rightarrow R'(0, 5) \\
S(-3, 0) & \rightarrow S'(0, 3) \\
T(-1, -3) & \rightarrow T'(3, 1)
\end{align*}
\]
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11. The translated image has endpoints \(X'(3, -3)\) and \(Y'(-4, -3)\). After the rotation of \(90^\circ\) about the origin, the endpoints are \(X'(-3, -3)\) and \(Y'(3, 4)\).

12. The rotation of \(180^\circ\) about the origin yields \(X'(3, -1)\) and \(Y'(-4, 5)\). The translation of \((x, y) \rightarrow (x - 1, y + 1)\) yields \(X''(2, 0)\) and \(Y''(-5, 6)\).

13. The rotation of \(270^\circ\) about the origin yields \(X'(1, 3)\) and \(Y'(-5, -4)\). The reflection in the y-axis yields \(X''(-1, 3)\) and \(Y''(5, -4)\).

14. The reflection in the line \(y = x\) yields \(X'(1, -3)\) and \(Y'(-5, 4)\). The rotation of \(180^\circ\) about the origin yields \(X'(-1, 3)\) and \(Y'(5, -4)\).

15. The rotation of \(90^\circ\) about the origin yields \(L'(-6, 1)\), \(M'(-4, -2)\), and \(N'(-2, 3)\). The translation \((x, y) \rightarrow (x - 3, y + 2)\) yields \(L''(-9, 3)\), \(M''(-7, 0)\), and \(N''(-5, 5)\).

16. The reflection in the x-axis yields \(L'(1, -6)\), \(M'(-2, -4)\), and \(N'(3, -2)\). The rotation of \(270^\circ\) about the origin yields \(L''(-6, -1)\), \(M''(-4, 2)\), and \(N''(-2, -3)\).

17. The rotations of \(90^\circ\) and \(180^\circ\) about the center will map this figure onto itself.

18. The rotations of \(72^\circ\) and \(144^\circ\) about the center will map this figure onto itself.

19. The rotations of \(45^\circ\), \(90^\circ\), \(135^\circ\), and \(180^\circ\) about the center will map this figure onto itself.

20. The rotation of \(180^\circ\) about the center will map this figure onto itself.

21. F; The angle of rotational symmetry of this figure is \(120^\circ\).

22. E and H; The angles of rotational symmetry of this figure are \(90^\circ\) and \(180^\circ\).

23. D and G; The angles of rotational symmetry of this figure are \(72^\circ\) and \(144^\circ\).

24. C, F, and H; The angles of rotational symmetry of this figure are \(60^\circ\), \(120^\circ\), and \(180^\circ\).

25. The rule for a \(270^\circ\) rotation should have been used instead of the rule for a reflection on the x-axis.

\((x, y) \rightarrow (y, -x)\)

\(C(-1, 1) \rightarrow C'(1, 1)\) instead of \(C'(-1, -1)\)

\(D(2, 3) \rightarrow D'(3, -2)\) instead of \(D'(2, -3)\)
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26. The rule for a 270° rotation should have been used instead of the rule for a reflection in the line y = x.

\( (x, y) \rightarrow (y, -x) \)

- \( C(-1, 1) \rightarrow C'(1, 1) \) instead of \( C'(-1, -1) \)
- \( D(2, 3) \rightarrow D'(3, -2) \) instead of \( D'(2, 3) \)

27. Draw and label \( \angle D \). Draw \( \triangle ABC \) and point \( O \). Placing the compass point on point \( D \), sweep an arc across \( D \)'s vectors.

Retain this compass setting. Connect \( O \) to \( A \) with a ray. Sweep an arc that will intersect ray \( OA' \). Connect \( O \) to \( B \) with a ray. Sweep an arc that will intersect line \( OB' \). Connect \( O \) to \( C \) with a ray. Sweep an arc that will intersect with ray \( OC' \). Use the compass to measure the distance of the arc vector intersections on \( \angle D \). Retain this setting with the point on \( A \) in order to sweep the arc for point \( A' \). Label point \( A' \). Put the unchanged compass point on \( A \). Place the unchanged compass point on \( B \). Sweep the arc for point \( B' \). Label point \( B' \). Put the unchanged compass point on \( C \). Sweep the arc for point \( C' \). Label point \( C' \). Connect \( A'B'C' \).

![Diagram of \( \triangle ABC \) and \( \angle D \)](diagram.png)

28. a. If you were outside, you are now inside, or vice versa, because you have made half of a rotation.

b. You are back where you started because you have made a full rotation.

29. a. The slope of the line rotated 90° clockwise is the opposite reciprocal of the slope of the preimage, and the \( y \)-intercept is equal to the \( x \)-intercept of the preimage. So, the equation of the line is \( y = \frac{-1}{2}x + \frac{3}{2} \).

The slope of the line rotated 180° is equal to the slope of the preimage, and the \( y \)-intercepts of the image and preimage are opposites. So, the equation of the line is \( y = 2x + 3 \).

The slope of the line rotated 270° clockwise is the opposite reciprocal of the slope of the preimage, and the \( y \)-intercept is the opposite of the \( x \)-intercept of the preimage. So, the equation of the line is \( y = \frac{1}{2}x - \frac{3}{2} \).

The equation of the line rotated 360° is the same as the equation of the preimage. So, the equation of the line is \( y = 2x - 3 \).

b. yes; Because the coordinates of every point change in the same way with each rotation, the relationships described will be true for an equation with any slope and \( y \)-intercept.

30. yes; Reflection in the \( y \)-axis and then in the \( x \)-axis yields \( (x, y) \rightarrow (x, -y) \rightarrow (-x, -y) \). A 180° rotation yields the same result: \( (x, y) \rightarrow (-x, -y) \).

31. If a figure only has point symmetry, then it has 180° rotational symmetry. One rotation will rotate the figure 180°. A second rotation will rotate the figure another 180°. The two rotations combined result in a 360° rotation, and therefore the rotations map the figure onto itself. So, you can rotate the figure twice before it is back where it started.

32. no; Because the figure has 90° rotational symmetry, the image will still be symmetrical to the preimage after two 90° rotations, which is the equivalent of a 180° rotation.

33. yes; Sample answer: A rectangle (that is not a square) is one example of a figure that has 180° rotational symmetry, but not 90° rotational symmetry.

34. yes; Sample answers:

- 90°: a reflection in \( y = x \) followed by a reflection in the \( y \)-axis: \( (x, y) \rightarrow (y, x) \rightarrow (-y, x) \)
- 180°: a reflection in the \( x \)-axis followed by a reflection in the \( y \)-axis: \( (x, y) \rightarrow (x, -y) \rightarrow (-x, -y) \)
- 270°: a reflection in \( y = x \) followed by a reflection in the \( x \)-axis: \( (x, y) \rightarrow (y, x) \rightarrow (y, -x) \)
- 360°: a reflection in the \( x \)-axis twice: \( (x, y) \rightarrow (x, -y) \rightarrow (x, y) \)

35. The number of lines of symmetry \( n \) times the measure of angle 1 equals 180°.

a. \( n(\angle 1) = 180° \)

b. \( n(\angle 1) = 180° \)

36. Rotating 90° clockwise will map point

\( P(a, b) \rightarrow P'(b, -a) \) just as in a 270° counterclockwise rotation.

Rotating 180° clockwise will map point

\( P(a, b) \rightarrow P'(−a, −b) \) just as in a 180° counterclockwise rotation.

Rotating 270° clockwise will map point

\( P(a, b) \rightarrow P'(b, a) \) just as in a 90° counterclockwise rotation.

37. Rotating \( \triangle XYZ \) 90° around point \( P \) yields

\( X(2, 5) \) → \( X'(−8, 3) \), \( Y(3, 1) \) → \( Y'(−4, 4) \), and \( Z(0, 2) \) → \( Z'(−5, 1) \).
Chapter 4

4.1 – 4.3 What Did You Learn? (p. 197)

4.1– 4.3 Quiz (p. 198)

Maintaining Mathematical Proficiency

40. Angles: ∠P ≅ ∠W, ∠Q ≅ ∠V, ∠R ≅ ∠Z, ∠S ≅ ∠Y.
   Sides: PQ ≅ WV, QR ≅ VZ, RS ≅ ZY, ST ≅ XY, TP ≅ WX

41. Angles: ∠A ≅ ∠I, ∠B ≅ ∠K, ∠C ≅ ∠L, ∠D ≅ ∠M
   Sides: AB ≅ JK, BC ≅ KL, CD ≅ LM, DA ≅ MJ

4.1–4.3 What Did You Learn? (p. 197)

1. Recreate the chess board on a coordinate plane and substitute
   the coordinates into your rule to verify that both the
   composition and the single translation yield the same result.

2. The x-coordinate is translated 3 units to the right, so add 3
   to x. The y-coordinate is translated 3 units up, so add 3 to y.

3. Find two points on the line y = 2x – 3 (e.g., the x- and
   y-intercepts), their images after the rotation, and use the
   images to find the equation of the new line.

4.1–4.3 Quiz (p. 198)

1. Using (x, y) → (x + 4, y – 2) yields A(−4, 1) → A’(0, −1),
   B(−3, 3) → B’(1, 1), C(0, 1) → C’(4, −1), and
   D(−2, 0) → D’(2, −2).

2. Using (x, y) → (x − 1, y − 5) yields
   A(−4, 1) → A’(−5, −4), B(−3, 3) → B’(−4, −2),
   C(0, 1) → C’(−1, −4), and D(−2, 0) → D’(−3, −5).

3. Using (x, y) → (x + 3, y + 6) yields A(−4, 1) → A’(−1, 7),
   B(−3, 3) → B’(0, 9), C(0, 1) → C’(3, 7), and
   D(−2, 0) → D’(1, 6).

4. Reflection in the x-axis yields A(−5, 6) → A’(−5, −6),
   B(−7, 8) → B’(−7, −8), and C(−3, 11) → C’(−3, −11).

5. Reflection in the line y = x yields D(−5, −1) → D’(−1, −5),
   E(−2, 1) → E’(1, −2), and F(−1, −3) → F’(−3, −1).
6. Reflection in the line \( x = 3 \) yields \( J(-1, 4) \rightarrow J'(7, 4) \), \( K(2, 5) \rightarrow K'(4, 5) \), \( L(5, 2) \rightarrow L'(1, 2) \), and \( M(4, -1) \rightarrow M'(2, -1) \).

7. Reflection in the line \( y = -2 \) yields \( P(2, -4) \rightarrow P'(2, 0) \), \( Q(6, -1) \rightarrow Q'(6, -3) \), \( R(9, -4) \rightarrow R'(9, 0) \), and \( S(6, -6) \rightarrow S'(6, 2) \).

8. The graph of the image of \( \triangle ABC \) after the translation \( (x, y) \rightarrow (x, y + 6) \) yields \( A(2, -1) \rightarrow A'(2, 5) \), \( B(5, 2) \rightarrow B'(5, 8) \), and \( C(8, -2) \rightarrow C'(8, 4) \). The reflection in the \( y \)-axis yields \( A'(2, 5) \rightarrow A''(-2, 5) \), \( B'(5, 8) \rightarrow B''(-5, 8) \), and \( C'(8, 4) \rightarrow C''(-8, 4) \).

9. The graph of the image of \( \triangle ABC \) after the translation \( (x, y) \rightarrow (x - 9, y) \) yields \( A(2, -1) \rightarrow A'(-7, -1) \), \( B(5, 2) \rightarrow B'(-4, 2) \), and \( C(8, -2) \rightarrow C'(-1, -2) \). The reflection in the line \( y = 1 \) yields \( A'(-7, -1) \rightarrow A''(-7, 3) \), \( B'(-4, 2) \rightarrow B''(-4, 0) \), and \( C'(-1, -2) \rightarrow C''(-1, 4) \).

10. There are 6 lines of symmetry.

11. There are no lines of symmetry.

12. There are 2 lines of symmetry.

13. There is 1 line of symmetry.

14. A \( 90^\circ \) rotation about the origin yields \( A(1, 1) \rightarrow A'(-1, 1) \), \( B(2, 4) \rightarrow B'(-4, 2) \), and \( C(4, 1) \rightarrow C'(-1, 4) \).

15. A \( 270^\circ \) rotation about the origin yields \( D(-3, 2) \rightarrow D'(2, 3) \), \( E(-1, 4) \rightarrow E'(4, 1) \), \( F(1, 2) \rightarrow F'(-1, -1) \), and \( G(1, -1) \rightarrow G'(-1, -1) \).
Chapter 4

16. A 180° rotation about the origin yields \( H(-4, 1) \rightarrow H'(4, -1), \)
\( I(-2, 2) \rightarrow I'(2, -2), J(-1, -2) \rightarrow J'(1, 2), \) and
\( K(-4, -4) \rightarrow K'(4, 4). \)

17. The translation \((x, y) \rightarrow (x - 4, y + 3)\) yields
\( L(-3, -2) \rightarrow L'(-7, 1), M(-1, 1) \rightarrow M'(-5, 4), \) and
\( N(2, -3) \rightarrow N'(-2, 0). \) The rotation of 180° about the origin yields
\( L'(-7, 1) \rightarrow L''(7, -1), M'(-5, 4) \rightarrow M''(5, -4), \) and
\( N'(-2, 0) \rightarrow N''(2, 0). \)

18. The rotation of 90° about the origin yields
\( L(-3, -2) \rightarrow L'(2, -3) \) (\( L' \) and \( N \) are the same),
\( M(-1, 1) \rightarrow M'(-1, -1), \) and \( N(2, -3) \rightarrow N'(3, 2). \) The reflection in the y-axis yields
\( L'(2, -3) \rightarrow L''(-2, -3), \)
\( M'(-1, -1) \rightarrow M''(1, -1), \) and \( N'(3, 2) \rightarrow N''(-3, 2). \)

19. Sample answer:
Step 1: Rotate orange figure 90° around point \((-2, 3).\)
Step 2: Translate orange figure 4 units right and 5 units down.
Step 3: Translate red figure 7 units down and 3 units right.
Step 4: Rotate purple figure 90° around point \((2, 3).\)
Step 5: Translate purple figure 3 units left and 7 units down.

4.4 Explorations (p. 199)
1. a. Check students’ work.
   b. Check students’ work.
   c. The line passes through points \( A, A', \) and \( A''. \)
   Sample answer:
   
   ![Diagram showing the line passing through points A, A', and A'']
   
   d. The distance between \( A \) and \( A'' \) is twice the distance between
   the parallel lines.
   e. yes; \( \triangle A'B''C'' \) is a translation of \( \triangle ABC. \)
   f. If two lines are parallel, and a preimage is reflected in
   the first line and then in the second, the final image is a
   translation of the preimage. The distance between each
   point in the preimage and its corresponding point in the
   final image is twice the distance between the parallel lines.

2. a. Check students’ work.
   b. Check students’ work.
   c. Sample answer: \( m \angle EDF = 50°; \) Rotating \( \triangle ABC \) about
   point \( D \) maps \( \triangle ABC \) onto \( \triangle A'B''C''. \)
   d. The final image after the reflections is the same as a
   rotation about point \( D \) using an angle that is twice the
   measure of the angle of intersection.
   e. The image of a figure reflected in two lines is congruent to
   the preimage. The image of a figure reflected in two parallel
   lines is a translation of the preimage. The image of a figure
   reflected in two lines that intersect in point \( D \) is a rotation in
   point \( D \) of the preimage.
   f. The distance of \( QQ'' \) is 2 times 3.2, or 6.4 inches.

4.4 Monitoring Progress (pp. 200–203)
1. For quadrilaterals \( IHGJ \) and \( QPNR \) to be congruent,
   corresponding angles and corresponding sides must be
   equal. \( IH = QP, HG = PN, GJ = NR, JI = RQ, \) and all
   angles are congruent because they are all 90° angles. So,
   \( \square IHGJ \equiv \square QPNR. \) \( \triangle LKM \) is a reflection in the y-axis of
   \( \triangle TSU. \) So, \( \triangle LKM \equiv \triangle TSU. \) \( \triangle DEF \) is a 90° rotation of
   \( \triangle ABC. \) So, \( \triangle DEF \equiv \triangle ABC. \)
Chapter 4

2. Sample answer: Translating quadrilateral $ABCD$ down 4 units yields $A(1, 1) \rightarrow A'(1, -3)$, $B(3, 1) \rightarrow B'(3, -3)$, $C(4, 3) \rightarrow C'(4, -1)$, and $D(2, 3) \rightarrow D'(2, -1)$. Reflecting over the $x$-axis yields $A'(1, -3) \rightarrow E(-1, -3)$, $D'(2, -1) \rightarrow H(-2, 1)$, $C'(4, -1) \rightarrow G(-4, -1)$, and $B'(3, -3) \rightarrow F(-3, -3)$.

3. Sample answer: Using the translation $(x, y) \rightarrow (x + 5, y)$ yields $L(-4, 2) \rightarrow L'(1, 2)$, $J(-1, 2) \rightarrow J'(4, 2)$, and $K(-3, 4) \rightarrow K'(2, 4)$. Reflecting $\triangle L'K'J'$ in the $y$-axis yields $L'(1, 2) \rightarrow P(1, -2)$, $J'(4, 2) \rightarrow M(4, -2)$, and $K'(2, 4) \rightarrow N(2, -4)$.

4. The transformation that maps the blue image to the green image is $(x, y) \rightarrow (x + 2, 3y)$. 

5. If $P'$ is the image of $P$ with a reflection in line $k$, then $PP' \perp k$ by the Reflections in Parallel Lines Theorem (Thm. 4.2).

6. The distance based on the Reflections in Parallel Lines Theorem (Thm. 4.2) is 2 times the distance between the two parallel lines, which is 3.2 centimeters.

7. Rotate the blue figure $160^\circ$ (2 \cdot 80) about $P$ in order to position the green figure.

8. The angle measures $76^\circ \div 2 = 38^\circ$.

4.4 Exercises (pp. 204–206)

Vocabulary and Core Concept Check
1. Two geometric figures are congruent if and only if there is a rigid motion or a composition of rigid motions that moves one of the figures onto the other.

2. The preimage and image are congruent in a rigid transformation.

Monitoring Progress and Modeling with Mathematics
3. $\triangle HJK$ is a $90^\circ$ rotation of $\triangle QRS$. So, $\triangle HJK \equiv \triangle QRS$. 

4. $\square DEFG$ is a translation 7 units right and 3 units down of $\square LMNP$. So, $\square DEFG \equiv \square LMNP$.

4. $\triangle MNP$ is a $90^\circ$ rotation of $\triangle TUV$. So, $\triangle MNP \equiv \triangle TUV$. 

5. $\triangle EFG$ is a $180^\circ$ rotation of $\triangle QRS$. So, $\triangle EFG \equiv \triangle QRS$. 

6. $\triangle HJKL$ is a translation 4 units down and 7 units right of $\square ABCD$. So, $\triangle HJKL \equiv \square ABCD$.

7. Apply the translation $(x, y) \rightarrow (x + 4, y)$ yields $Q(2, 4) \rightarrow T(6, 4)$, $R(5, 4) \rightarrow U(9, 4)$, and $S(4, 1) \rightarrow V(8, 1)$. So, $\triangle QRS \equiv \triangle TUV$.

8. Rotating quadrilateral $WXYZ$ $90^\circ$ yields $CDEF$. So, $WXYZ \equiv CDEF$.

9. $M$ and $N$ are translated 2 units right of their corresponding vertices, $L$ and $K$, but $P$ is translated only 1 unit right of its corresponding vertex, $J$. So, this is not a rigid motion.

10. Translate $ABCD$ 5 units down, followed by a reflection in the $y$-axis. So, $ABCD \equiv GHEF$.

11. The translation maps $\triangle ABC$ onto $\triangle A'B'C''$.

12. Lines $k$ and $m$ are perpendicular to $AA''$.

13. The length of $CC''$ is $2 \cdot 2.6 = 5.2$ inches.

14. yes; Because $\triangle A'B'C''$ is a reflection of $\triangle A'B'C'$ in line $m$, each vertex in the image is the same distance from the line of reflection as its preimage.

15. The angle of rotation that maps $A$ to $A''$ is $110^\circ$.

16. The angle of rotation that maps $A$ to $A''$ is $30^\circ$.

17. The translation $(x, y) \rightarrow (x + 5, y)$ maps $\triangle ABC$ to $\triangle A'B'C'$: $A(-4, 1) \rightarrow A'(1, 1)$, $B(-4, 2) \rightarrow B'(1, 2)$, and $C(-1, 1) \rightarrow C'(4, 1)$. A reflection in the $x$-axis maps $\triangle A'B'C'$ to $\triangle A''B''C''$: $A'(1, 1) \rightarrow A''(-1, 1)$, $B'(1, 2) \rightarrow B''(1, -2)$, and $C'(4, 1) \rightarrow C''(4, -1)$.

18. According to the Reflections in Intersecting Lines Theorem (Thm. 4.3), the angle of rotation is $2x^\circ$, where $x^\circ$ is the measure of the acute angle formed by the two intersecting lines. The angle of rotation is $2(72^\circ) = 144^\circ$.

19. The angle formed by two intersecting lines with a measure of $42^\circ$ yields a rotation of $84^\circ$ that will map $C$ to $C'$.

20. The angle formed by two intersecting lines with a measure of $12^\circ$ yields a rotation of $24^\circ$ that will map $C$ to $C'$.

21. The angle formed by two intersecting lines with a measure of $90^\circ$ yields a rotation of $(x, y) \rightarrow (-x, -y)$ that will map $C$ to $C'$.

22. The angle formed by two intersecting lines with a measure of $45^\circ$ yields a rotation of $(x, y) \rightarrow (y, -x)$ that will map $C$ to $C'$.

23. Consecutive reflections in each of two parallel lines is equivalent to a translation. A reflection in a third line is equivalent to a glide transformation.

24. a. yes; All of the figures could be created using one or more rigid transformations of an original shape.
25. never; A congruence transformation is a rigid motion that preserves length and angle measurement.

26. always; Every figure can be mapped onto a congruent figure using transformations.

27. sometimes; Reflecting in $y = x$ and then $y = x$ is not a rotation. Reflecting in the $y$-axis and then the $x$-axis is a rotation of $180^\circ$.

28. sometimes; It would depend on the translations.

29. no; The preimage is smaller than the projected image.

30. a. Triangle 5 is congruent to Triangle 8 by a translation or reflections in parallel lines.

b. Triangle 1 is congruent to Triangle 4 by a reflection.

c. Triangle 2 is congruent to Triangle 7 by a rotation or reflections in intersecting lines.

d. Pentagon 3 is congruent to Pentagon 6 by a glide reflection or two reflections in parallel lines and then a reflection in a perpendicular line.

31.

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. A reflection in line $\ell$ maps $JK$ to $J''K''$, a reflection in line $m$ maps $J''K''$ to $JK''$, and $\ell \parallel m$.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. If $KK''$ intersects line $\ell$ at $L$ and line $m$ at $M$, then $L$ is the perpendicular bisector of $KK''$, and $M$ is the perpendicular bisector of $K''K'$.</td>
<td>2. Definition of reflection</td>
</tr>
<tr>
<td>3. $KK''$ is perpendicular to $\ell$ and $m$, and $KL = LK'$ and $K'M = MK''$.</td>
<td>3. Definition of perpendicular bisector</td>
</tr>
<tr>
<td>4. If $d$ is the distance between $\ell$ and $m$, then $d = LM$.</td>
<td>4. Ruler Postulate</td>
</tr>
<tr>
<td>5. $LM = LK' + K'M$ and $KK'' = KL + LK' + K'M + MK''$</td>
<td>5. Segment Addition Postulate</td>
</tr>
<tr>
<td>6. $KK'' = 2(LK' + K'M)$</td>
<td>6. Substitution Property of Equality</td>
</tr>
<tr>
<td>7. $KK'' = 2LM$</td>
<td>7. Distributive Property</td>
</tr>
<tr>
<td>8. $KK'' = 2d$</td>
<td>8. Substitution Property of Equality</td>
</tr>
</tbody>
</table>

32. Sample answer:

Translations and rotations are used.

33. The second classmate is correct. A translation reverses the segment endpoints. A rotation retains the mapping of the segments and points. Reflecting in the $y$-axis yields $P(1, 3) \rightarrow P'(-1, 3)$ and $Q(3, 2) \rightarrow Q'(-3, 2)$. Reflecting in the $x$-axis yields $P'(-1, 3) \rightarrow P''(-1, -3)$ and $Q'(-3, 2) \rightarrow Q''(-3, -2)$.

Using the translation $(x, y) \rightarrow (x - 4, y - 5)$ yields $P(1, 3) \rightarrow P'(-3, -2)$ and $Q(3, 2) \rightarrow Q'(-1, -3)$.

A rotation of $180^\circ$ yields $P(1, 3) \rightarrow P'(-1, -3)$ and $Q(3, 2) \rightarrow Q'(-3, -2)$.

34. yes; no; Reflecting in line $m$ first maps the final triangle to a spot to the left of line $m$. Reflecting in line $\ell$ first maps the final triangle to a spot to the right of line $m$.

35. Reflect $\triangle ABC$ over line $m$. Then reflect over a line $\ell$ parallel to line $m$ to form $\triangle A'B'C''$.

36. 

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37. $5x + 16 = -3x$

$16 = -8x$

$x = -2$

38. $12 + 6m = 2m$

$12 = -4m$

$m = -3$

39. $4b + 8 = 6b - 4$

$-2b = -12$

$b = 6$
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40. $7w - 9 = 13 - 4w$
   
   $11w = 22$
   
   $w = 2$

41. $7(2n + 11) = 4n$
   
   $14n + 77 = 4n$
   
   $77 = -10n$
   
   $n = \frac{-77}{10}$

42. $-2(8 - y) = -6y$
   
   $-16 + 2y = -6y$
   
   $-16 = -8y$
   
   $y = 2$

43. Increase: $625 - 500 = 125$

   $125 = 500 \cdot x$
   
   $x = \frac{125}{500}$
   
   $x = 0.25$

   The percent of increase is 25%.

4.5 Explorations (p. 207)

1. a. Check students’ work. The $x$-value of each vertex of $\triangle A'B'C'$ is twice the $x$-value of its corresponding vertex of $\triangle ABC$, and the $y$-value of each vertex of $\triangle A'B'C'$ is twice the $y$-value of its corresponding vertex of $\triangle ABC$. Each side of $\triangle A'B'C'$ is twice as long as its corresponding side of $\triangle ABC$. Each angle of $\triangle A'B'C'$ is congruent to its corresponding angle of $\triangle ABC$.

   b. The $x$-value of each vertex of $\triangle A'B'C'$ is half of the $x$-value of its corresponding vertex of $\triangle ABC$, and the $y$-value of each vertex of $\triangle A'B'C'$ is half of the $y$-value of its corresponding vertex of $\triangle ABC$. Each side of $\triangle A'B'C'$ is half as long as its corresponding side of $\triangle ABC$. Each angle of $\triangle A'B'C'$ is congruent to its corresponding angle of $\triangle ABC$.

   c. The $x$-value of each vertex of $\triangle A'B'C'$ is $k$ times the $x$-value of its corresponding vertex of $\triangle ABC$, and the $y$-value of each vertex of $\triangle A'B'C'$ is $k$ times the $y$-value of its corresponding vertex of $\triangle ABC$. Each side of $\triangle A'B'C'$ is $k$ times as long as its corresponding side of $\triangle ABC$. Each angle of $\triangle A'B'C'$ is congruent to its corresponding angle of $\triangle ABC$.

   d. The image is a line that coincides with $\overline{AB}$.

   e. The image is a line that is parallel to $\overline{AC}$. The $x$- and $y$-intercepts of the image are each three times the $x$- and $y$-intercepts of $\overline{AC}$.

   f. The image of $\overline{AC}$ is a line that is parallel to $\overline{AC}$. The $x$- and $y$-intercepts of the image are each one-fourth of the $x$- and $y$-intercepts of $\overline{AC}$.

   g. When you dilate an image that passes through the center of dilation, the image coincides with the preimage. When you dilate an image that does not pass through the center of dilation, the image is parallel to the preimage, and the image has intercepts that can be found by multiplying the intercepts of the preimage by the constant of dilation.

   h. To reduce or enlarge a figure, the image is proportional to the preimage.

   i. The difference between the $x$-value of each vertex of $\triangle A'B'C'$ and the $x$-value of the center of dilation is equal to $k$ times the difference between its corresponding $x$-value of $\triangle ABC$ and the $x$-value of the center of dilation. The difference between the $y$-value of each vertex of $\triangle A'B'C'$ and the $y$-value of the center of dilation is equal to $k$ times the difference between its corresponding $y$-value of $\triangle ABC$ and the $y$-value of the center of dilation. Each side of $\triangle A'B'C'$ is $k$ times as long as its corresponding side of $\triangle ABC$. Each angle of $\triangle A'B'C'$ is congruent to its corresponding angle of $\triangle ABC$.

   Sample answers:
4.5 Monitoring Progress (pp. 208–211)

1. The scale factor is \( \frac{3}{12} = \frac{1}{4} \). The dilation is a reduction because the scale factor is less than 1.

2. \((x, y) \rightarrow (4x, 4y)\): \(P(-2, -1) \rightarrow P'(-8, -4)\), \(Q(-1, 0) \rightarrow Q'(-4, 0)\), \(R(0, -1) \rightarrow R'(0, -4)\)

3. \((x, y) \rightarrow (0.4x, 0.4y)\): \(P(5, -5) \rightarrow P'(2, -2)\), \(Q(10, -5) \rightarrow Q'(4, -2)\), \(R(10, 5) \rightarrow R'(4, 2)\)

4. \((x, y) \rightarrow (-2x, -2y)\): \(P(1, 2) \rightarrow P'(-2, -4)\), \(Q(3, 1) \rightarrow Q'(-6, -2)\), \(R(1, -3) \rightarrow R'(-2, 6)\)

5. According to the Coordinate Rule for Dilations, if the origin \(P(0, 0)\) is the preimage of a point, then its image after a dilation centered at the origin with a scale factor \(k\) is the point \(P'(k \cdot x, k \cdot y)\), which is also the origin, or \((0, 0)\).

6. The scale factor of the dilation is \(\frac{8}{4.5} \approx \frac{80}{45} = \frac{16}{9}\), 16 to 9.

7. \(\frac{12.6}{x} = 6\)
   
   \[12.6 = 6x\]
   
   \[\frac{12.6}{6} = x\]
   
   \[x = 2.1\]

   The actual length of the spider is 2.1 centimeters.

4.5 Exercises (pp. 212–214)

Vocabulary and Core Concept Check

1. If \(P(x, y)\) is the preimage of a point, then its image after a dilation centered at the origin \((0, 0)\) with scale factor \(k\) is the point \(P'(kx, ky)\).

2. The scale factor that does not belong is 60%. The scale factors of the others are: \(\frac{5}{4} > 1\), 115% = \(\frac{115}{100} = \frac{23}{20}\) > 1, and 2 > 1, which are all enlargements. The scale factor 60% = \(\frac{60}{100} = \frac{3}{5}\) is a reduction.

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3. The scale factor of \(P\) to \(P'\) is \(\frac{5}{12} = \frac{5}{12}\), which is a reduction.

4. The scale factor of \(P\) to \(P'\) is \(\frac{24}{8} = \frac{3}{1}\), which is an enlargement.

5. The scale factor of \(P\) to \(P'\) is \(\frac{9}{15} = \frac{3}{5}\), which is a reduction.

6. The scale factor of \(P\) to \(P'\) is \(\frac{28}{8} = \frac{7}{2}\), which is an enlargement.

7. Not drawn to scale.

Construct lines \(\overline{CM}, \overline{CN}\), and \(\overline{CL}\). Measure the distance between \(C\) and \(M\). Keeping the same compass setting, place the compass point on \(M\) and sweep \(\overline{CM}\), placing \(M'\) at the intersection of the arc and the line. Do the same for points \(C\) to \(N\) and \(C\) to \(L\). Connect \(M', N',\) and \(L'\) to form \(\triangle L'M'N'\).

8. Not drawn to scale.

Construct lines \(\overline{PM}, \overline{PN}\), and \(\overline{PL}\). Measure the distance between \(C\) and \(M\). Keeping the same compass setting, place the compass point on \(M\) and sweep the line. Then place the compass point onto the arc and sweep a second arc on \(\overline{PM}\). Place \(M'\) at the intersection of the arc and the line. Do the same for points \(C\) to \(N\) and \(C\) to \(L\). Connect \(L', M',\) and \(N'\) to form \(\triangle L'M'N'\).
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9. **Not drawn to scale.**

   Bisect the two sides with vertex \( M \). Connect the midpoints of the two bisected sides to form the third side.

10. **Not drawn to scale.**

    Construct lines \( \overline{CM}, \overline{CN}, \overline{CL} \). Measure the distance between \( C \) and \( M \). Determine what \( \frac{1}{4} \) the distance is and adjust the compass setting. Place the compass point on \( M \) and sweep \( \overline{CM} \), placing \( M' \) at the intersection of the arc and the line. Do the same for points \( C \) to \( N \) and \( C \) to \( L \). Connect \( M', N', \) and \( L' \) to form \( \triangle L'M'N' \).

11. **Not drawn to scale.**

    Construct lines \( \overline{CS}, \overline{CR}, \overline{CT}, \overline{CU} \), and \( \overline{CP} \). Measure the distance between \( C \) and \( S \). Keeping the same compass setting, place the compass point onto the arc and sweep a second arc on \( \overline{CS} \). Place \( S' \) at the intersection of the arc and the line. Do the same for points \( C \) to \( R \), \( C \) to \( T \), \( C \) to \( U \), and \( C \) to \( P \). Connect \( R', S', T', \) and \( U' \) to form the image of \( \triangle RSTU \).

12. **Not drawn to scale.**

    Construct lines \( \overline{PR}, \overline{PS}, \overline{PT}, \) and \( \overline{PU} \). Measure the distance between \( P \) and \( R \). Determine what \( \frac{1}{4} \) the distance is and adjust the compass setting. Place the compass point on \( P \) and sweep \( \overline{PR} \), placing \( R' \) at the intersection of the arc and the line. Do the same for points \( P \) to \( S \), \( P \) to \( T \), and \( P \) to \( U \). Connect \( R', S', T', \) and \( U' \) to form \( \triangle R'S'T'U' \).

13. **Not drawn to scale.**

    Measure the distance between \( R \) and \( U \), and determine \( \frac{1}{4} \) of that distance. Place \( U' \) at that distance from \( R \). Measure the distance from \( R \) to \( S \), and determine \( \frac{1}{4} \) of that distance and place the point \( S' \). Do the same for \( T \) and \( P \). Place \( P' \) in the appropriate position. Then connect \( U', R', S', \) and \( T \) to form the dilated image of \( \triangle URST \).

14. **Not drawn to scale.**

    Construct lines \( \overline{CR}, \overline{CS}, \overline{CT}, \) and \( \overline{CU} \). Measure the distance between \( C \) and \( R \). Determine what \( \frac{1}{4} \) the distance is and adjust the compass setting. Place the compass point on \( R \) and sweep \( \overline{CR} \), placing \( R' \) at the intersection of the arc and the line. Do the same for points \( C \) to \( S \), \( C \) to \( T \), and \( C \) to \( U \). Connect \( R', S', T', \) and \( U' \) to form \( \triangle RSTU \).
Chapter 4

15. \((x, y) \rightarrow (3x, 3y)\)
   - \(X(6, -1) \rightarrow X'(18, -3)\)
   - \(Y(-2, -4) \rightarrow Y'(-6, -12)\)
   - \(Z(1, 2) \rightarrow Z'(3, 6)\)

16. The scale factor is \(120\% = \frac{120}{100} = \frac{6}{5}\).
   - \((x, y) \rightarrow \left(\frac{6}{5}x, \frac{6}{5}y\right)\)
   - \(A(0, 5) \rightarrow A'(0, 6)\)
   - \(B(-10, -5) \rightarrow B'(-12, -6)\)
   - \(C(5, -5) \rightarrow C'(6, -6)\)

17. \((x, y) \rightarrow \left(\frac{2}{3}x, \frac{2}{3}y\right)\)
   - \(T(9, -3) \rightarrow T'(6, -2)\)
   - \(U(6, 0) \rightarrow U'(4, 0)\)
   - \(V(3, 9) \rightarrow V'(2, 6)\)
   - \(W(0, 0) \rightarrow W'(0, 0)\)

18. \((x, y) \rightarrow \left(\frac{1}{4}x, \frac{1}{4}y\right)\)
   - \(J(4, 0) \rightarrow J'(1, 0)\)
   - \(K(-8, 4) \rightarrow K'(-2, 1)\)
   - \(L(0, -4) \rightarrow L'(0, -1)\)
   - \(M(12, -8) \rightarrow M'(3, -2)\)

19. \((x, y) \rightarrow \left(-\frac{1}{5}x, -\frac{1}{5}y\right)\)
   - \(B(-5, -10) \rightarrow B'(1, 2)\)
   - \(C(-10, 15) \rightarrow C'(2, -3)\)
   - \(D(0, 5) \rightarrow D'(0, -1)\)

20. \((x, y) \rightarrow (-3x, -3y)\)
   - \(M(-4, 1) \rightarrow M'(12, -3)\)
   - \(N(-3, -6) \rightarrow N'(9, 18)\)
   - \(L(0, 0) \rightarrow L'(0, 0)\)
Chapter 4

21. \((x, y) \rightarrow (-4x, -4y)\)
   \[R(-7, -1) \rightarrow R'(28, 4)\]
   \[S(2, 5) \rightarrow S'(-8, -20)\]
   \[T(-2, -3) \rightarrow T'(8, 12)\]
   \[U(-3, -3) \rightarrow U'(12, 12)\]

22. \((x, y) \rightarrow \left(\frac{-1}{2}x, \frac{-1}{2}y\right)\)
   \[W(8, -2) \rightarrow W'(-4, 1)\]
   \[X(6, 0) \rightarrow X'(-3, 0)\]
   \[Y(-6, 4) \rightarrow Y'(3, -2)\]
   \[Z(-2, 2) \rightarrow Z'(1, -1)\]

23. The scale factor should be calculated by finding \(\frac{CP'}{CP}\), not \(\frac{CP}{CP'}\). So, \(k = \frac{3}{12} = \frac{1}{4}\).

24. The scale factor should be calculated by finding the ratio of the length of a side of the image to the length of the corresponding side of the preimage; \(k = \frac{3}{2} = 2\).

25. Scale factor of the dilation: \(\frac{15}{9} = \frac{5}{3}\)
   \[\frac{5}{3} = \frac{35}{x}\]
   \[5x = 3 \cdot 35\]
   \[x = \frac{105}{5}\]
   \[x = 21\]
   The scale factor of the dilation is \(\frac{5}{3}\), and \(x = 21\).

26. Scale factor of the dilation: \(\frac{28}{14} = 2\)
   \[\frac{2}{1} = \frac{12}{n}\]
   \[2n = 12\]
   \[n = \frac{12}{2}\]
   \[n = 6\]
   The scale factor of the dilation is 2, and \(n = 6\).

27. Scale factor of the dilation: \(\frac{2}{3}\)
   \[\frac{2}{y} = \frac{2}{3}\]
   \[2 \cdot 3 = 2 \cdot y\]
   \[6 = 2y\]
   \[y = 3\]
   The scale factor of the dilation is \(\frac{2}{3}\), and \(y = 3\).

28. Scale factor of the dilation: \(\frac{7}{28} = \frac{1}{4}\)
   \[\frac{1}{4} = \frac{m}{4}\]
   \[1 \cdot m = 4 \cdot 4\]
   \[m = 16\]
   The scale factor of the dilation is \(\frac{1}{4}\), and \(m = 16\).

29. \(\frac{5}{2.5} = \frac{5}{\left(\frac{25}{10}\right)} = \left(\frac{5}{1}\right) \cdot \left(\frac{10}{25}\right) = \frac{50}{25} = 2\)
   The scale factor of this dilation is 2.

30. \(\frac{10}{8.5} = \frac{\left(\frac{10}{1}\right)}{\left(\frac{85}{10}\right)} = \frac{10}{85} = \frac{100}{85} = \frac{20}{17}\)
   The scale factor of this dilation is \(\frac{20}{17}\).

31. The length of the image of the emperor moth is \(5 \cdot 60 = 300\) millimeters.

32. The length of the image of the ladybug is \(10 \cdot 4.5 = 45\) millimeters.

33. The length of the image of the dragonfly is \(20 \cdot 47 = 940\) millimeters.

34. The length of the image of the carpenter ant is \(15 \cdot 12 = 180\) millimeters.
35. The grasshopper has a scale factor of 7.5.
\[
\frac{\text{magnified}}{\text{actual}} = \frac{15}{2} = 7.5
\]
The grasshopper has a scale factor of 7.
\[
\frac{\text{magnified}}{\text{actual}} = \frac{\left(\frac{42}{10}\right)}{\left(\frac{6}{10}\right)} = \frac{42}{10} \cdot \frac{10}{6} = \frac{42}{6} = 7
\]
The black beetle has a scale factor of 7.5.
\[
\frac{\text{magnified}}{\text{actual}} = \frac{\left(\frac{25}{16}\right)}{\left(\frac{8}{5}\right)} = \frac{15}{2} = 7.5
\]
The honey bee has a scale factor of 7.5.
\[
\frac{\text{magnified}}{\text{actual}} = \frac{\left(\frac{29.25}{10}\right)}{\left(\frac{39}{10}\right)} = \frac{2925}{10} \cdot \frac{10}{39} = \frac{75}{10} = 7.5
\]
The monarch butterfly has a scale factor of 7.5.

36. Sample answer:

The origin (0, 0) is the center of dilation. After drawing \(\triangle ABC\) and its dilation, draw the lines connecting each vertex in the preimage with its corresponding vertex in the image. These three lines intersect at the center of dilation.

37. No; The scale factor for the shorter sides is \(\frac{8}{4} = 2\), but the scale factor for the longer sides is \(\frac{10}{6} = \frac{5}{3}\). The scale factor for both sides has to be the same or the picture will be disturbed.

38. With a scale factor of \(\frac{1}{3}\), this indicates a reduction because \(\frac{1}{3} < 1\). So, the original figure is the larger star and the dilated figure is the smaller star.

39. The scale factor is \(\frac{6}{2} = 3\).
\[
3(x + 1) = 2x + 8
\]
\[
x + 3 = 2x + 8
\]
\[
x = 5
\]
In dilated figures, corresponding angles are equal.
\[
y + 16 = 3y - 34
\]
\[
16 = 2y - 34
\]
\[
50 = 2y
\]
\[
y = 25
\]
So, \(x = 5\) and \(y = 25\).

40. Change 200% to a fraction: \(\frac{200}{100} = \frac{2}{1} = 2\). So, a figure that is 200% larger than the preimage will be twice as large.

41. With a scale factor of 3, this indicates that the dilated figure is larger than the original (3 > 1). So, the original figure is closer to the center of dilation, which is inside both.

42. With a scale factor of \(\frac{1}{3}\), this indicates that the dilated figure is smaller than the original (\(\frac{1}{3} < 1\)). So, the dilated figure is closer to the center of dilation, which is inside the original figure.

43. With a scale factor of 120% (\(\frac{120}{100} = \frac{6}{5} = 1.2\)), this indicates that the dilated figure is 1.2 times larger than the original (1.2 > 1). So, the original figure is closer to the center of dilation, which is outside both.

44. With a scale factor of 0.1 (\(\frac{1}{10}\)), this indicates that the dilated figure is smaller than the original (\(\frac{1}{10} < 1\)). So, the dilated figure is closer to the center of dilation, which is outside both.

45. a. \(\overline{OA}\) is half of \(\overline{O'A'}\). or \(\overline{O'A'}\) is twice the length of \(\overline{OA}\).

b. \(\overline{O'A'}\) and \(\overline{OA}\) are the same line.
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46. \( \overline{A'B'} \) is half the length of \( \overline{AB} \).

b. \( \overline{A'B'} \parallel \overline{AB} \) and the y-intercept of \( \overline{A'B'} \) is half of the y-intercept of \( \overline{AB} \).

47. Change 11 feet to 11 \( \cdot \) 12 = 132 inches and 12 feet to 12 \( \cdot \) 12 = 144 inches.

132\( k = \frac{825}{100} \)

132\( k = \frac{33}{4} \)

\( k = \frac{33}{132} \cdot \frac{1}{4} = \frac{11}{44} = \frac{1}{16} \)

144\( k = 9 \)

\( k = \frac{9}{144} = \frac{1}{16} \)

The scale factor that dilates the room to the blue print is \( \frac{1}{16} \).

48. no; It is true that dilating a figure with a scale factor of 1 will not enlarge or reduce the image nor will \(-1\). However, by dilating with a scale factor that is negative, the image is rotated by 180°.

49. a. Find the length and width of the rectangle.

Length: \( XY = | -3 - 5 | = |-8| = 8 \)

Width: \( YZ = | 3 - (-1) | = | 4 | = 4 \)

\( P = 2 \ell + 2w = 2(8) + 2(4) = 16 + 8 = 24 \text{ units} \)

\( A = \ell w = 8 \cdot 4 = 32 \text{ square units} \)

b. \( (x, y) \rightarrow (3x, 3y) \)

\( W(-3, -1) \rightarrow W'(-9, -3) \)

\( X(-3, 3) \rightarrow X'(-9, 9) \)

\( Y(5, 3) \rightarrow Y'(15, 9) \)

\( Z(5, -1) \rightarrow Z'(15, -3) \)

Length: \( X'Y' = | -9 - 15 | = |-24| = 24 \)

Width: \( Y'Z' = | 9 - (-3) | = | 12 | = 12 \)

\( P = 2 \ell + 2w = 2(24) + 2(12) = 72 \text{ units} \)

\( A = \ell w = 24 \cdot 12 = 288 \text{ square units} \)

The perimeter of the dilated rectangle is three times the perimeter of the original rectangle. The area of the dilated rectangle is nine times the area of the original rectangle.

c. \( (x, y) \rightarrow \left( \frac{1}{2}x, \frac{1}{2}y \right) \)

\( W(-3, -1) \rightarrow W'(-\frac{3}{4}, -\frac{1}{2}) \)

\( X(-3, 3) \rightarrow X'(-\frac{3}{4}, \frac{3}{2}) \)

\( Y(5, 3) \rightarrow Y'(\frac{5}{2}, \frac{3}{2}) \)

\( Z(5, -1) \rightarrow Z'(\frac{5}{2}, -\frac{1}{2}) \)

Length: \( X'Y' = \left| -\frac{3}{4} - \frac{3}{2} \right| = \left| -\frac{9}{4} \right| = \frac{9}{4} = 2 \)

Width: \( Y'Z' = \left| -\frac{3}{4} - (-\frac{1}{2}) \right| = \left| -\frac{1}{4} \right| = 1 \)

\( P = 2 \ell + 2w = 2(2) + 2(1) = 6 \text{ units} \)

\( A = \ell w = 2 \cdot 1 = 2 \text{ square units} \)

The perimeter of the dilated rectangle is \( \frac{1}{2} \) the perimeter of the original rectangle. The area of the dilated rectangle is \( \frac{1}{16} \) the area of the original rectangle.

d. When a figure is dilated, the perimeter changes by a factor of \( k \). The area changes by a factor of \( k^2 \).

50. The center of dilation must be on that page. So, this point will be in the same place for both the original figure and the dilated figure.

51. With the center \((4, 0)\) and a scale factor of \(2\), the coordinates of the vertices are

\( A(4, 2) \rightarrow A'(4, 4) \), \( B(4, 6) \rightarrow B'(4, 12) \), and \( C(7, 2) \rightarrow C'(10, 4) \).

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52. The coordinates of the vertices of \( \triangle A'B'C' \) after the translation \((x, y) \rightarrow (x, y - 4)\) are \( A'(2, -5) \), \( B'(0, 0) \), and \( C'(-3, 1) \).

53. The coordinates of the vertices of \( \triangle A'B'C' \) after the translation \((x, y) \rightarrow (x - 1, y + 3)\) are \( A'(1, 2) \), \( B'(-1, 7) \), and \( C'(-4, 8) \).

54. The coordinates of the vertices of \( \triangle A'B'C' \) after the translation \((x, y) \rightarrow (x + 3, y - 1)\) are \( A'(5, -3) \), \( B'(3, 3) \), and \( C'(0, 4) \).

55. The coordinates of the vertices of \( \triangle A'B'C' \) after the translation \((x, y) \rightarrow (x - 2, y)\) are \( A'(0, -1) \), \( B'(-2, 4) \), and \( C'(-5, 5) \).

56. The coordinates of the vertices of \( \triangle A'B'C' \) after the translation \((x, y) \rightarrow (x + 1, y - 2)\) are \( A'(3, -3) \), \( B'(1, 2) \), and \( C'(-2, 3) \).

57. The coordinates of the vertices of \( \triangle A'B'C' \) after the translation \((x, y) \rightarrow (x - 3, y + 1)\) are \( A'(-1, 0) \), \( B'(-3, 5) \), and \( C'(-6, 6) \).
4.6 Explorations (p. 215)

1. a. Check students’ work.
   b. Check students’ work. Each side of \( \triangle A'B'C' \) is three times as long as its corresponding side of \( \triangle ABC \). The corresponding angles are congruent. Because the corresponding sides are proportional and the corresponding angles are congruent, the image is similar to the original triangle.

2. a. Sample answer:

   ![Sample answer diagram]

   yes; Because the corresponding sides are congruent and the corresponding angles are congruent, the image is similar to the original triangle.

   b. Sample answer:

   ![Sample answer diagram]

   yes; Because the corresponding sides are congruent and the corresponding angles are congruent, the image is similar to the original triangle.

3. yes; Translations, reflections, and rotations preserve side lengths and angle measurements. Dilations preserve angle measurements. Images are the same shape as the preimages and the corresponding angle measurements are equal.

4. yes; According to Composition Theorem (Thm. 4.1), the composition of two or more rigid motions is a rigid motion. Also, a dilation preserves angle measures and results in an image with lengths proportional to the preimage lengths. So, a composition of rigid motions or dilations will result in an image that has angle measures congruent to the corresponding angle measures of the original figure, and sides that are either congruent or proportional to the corresponding sides of the original figure.

4.6 Monitoring Progress (pp. 216–218)

1. Rotation: \( C(-2, 2) \rightarrow C'(-2, -2); D(2, 2) \rightarrow D'(-2, 2) \)
   Dilation: \( C'(-2, -2) \rightarrow C''(-1, -1); D'(-2, 2) \rightarrow D''(-1, 1) \)

   ![Rotation and Dilation diagrams]

2. Reflection in the x-axis: \( F(1, 2) \rightarrow F'(1, -2), G(4, 4) \rightarrow G'(4, -4), H(2, 0) \rightarrow H'(2, 0) \)
   Dilation: \( F'(1, -2) \rightarrow F''(1.5, -3); G'(4, -4) \rightarrow G''(6, -6), H'(2, 0) \rightarrow H''(3, 0) \)

   ![Reflection and Dilation diagrams]
3. **Sample answer:** Reflect $PQRS$ in the $x$-axis, and then dilate with a scale factor of $-\frac{1}{3}$.

   Reflection:
   \[ P(-6, 3) \rightarrow P'(6, -3), \quad Q(-3, 3) \rightarrow Q'(-3, -3), \quad R(0, -3) \rightarrow R'(0, 3), \quad S(-6, -3) \rightarrow S'(6, -3) \]

   Dilation: $P'(-6, -3) \rightarrow W(2, 1), \quad Q'(-3, -3) \rightarrow X(1, 1), \quad R'(0, 3) \rightarrow Y(0, -1), \quad S'(6, -3) \rightarrow Z(2, -1)$

4. **Sample answer:** Dilate $DEFG$ with a scale factor of $\frac{1}{2}$, and then rotate $180^\circ$ about the origin.

   Dilation:
   \[ D(-2, 3) \rightarrow D'(-1, \frac{3}{2}), \quad E(1, 4) \rightarrow E'\left(\frac{1}{2}, 2\right), \quad F(4, 0) \rightarrow F'(2, 0), \quad G(0, 1) \rightarrow G'\left(0, \frac{1}{2}\right) \]

   Rotation: $D'(-1, \frac{3}{2}) \rightarrow S\left(1, -\frac{3}{2}\right), \quad E'\left(\frac{1}{2}, 2\right) \rightarrow T\left(-\frac{1}{2}, -2\right), \quad F'(2, 0) \rightarrow U(-2, 0), \quad G'\left(0, \frac{1}{2}\right) \rightarrow V(0, -\frac{1}{2})$

5. **Given** Right isosceles $\triangle JKL$, with leg length $t$

   Right isosceles $\triangle MNP$, with leg length $v$

   Prove $\triangle JKL$ is similar to $\triangle MNP$.

   Translate $\triangle JKL$ so that point $L$ maps to point $P$. Because translations map segments to parallel segments and $\overline{LJ} \parallel \overline{PM}$, the image of $\overline{LJ}$ lies on $\overline{PM}$.

   Because translations preserve side lengths and angle measures, the image of $\triangle JKL$, $\triangle J'K'P'$, is a right isosceles triangle with leg length $t$. Because $\angle K'JP'$ and $\angle NPM$ are right angles, they are congruent. When $\overline{PJ'}$ coincides with $\overline{PM}$, $\overline{PK'}$ coincides with $\overline{PN}$. So, $\overline{PK'}$ lies on $\overline{PN}$. Next, dilate $\triangle J'K'P'$ using center of dilation $P$. Choose the scale factor to be the ratio of the side lengths of $\triangle MNP$ and $\triangle J'K'P'$, which is $\frac{v}{t}$.

   The dilation maps $\overline{PJ'}$ to $\overline{PM}$ and $\overline{PK'}$ to $\overline{PN}$ because the images of $\overline{PJ'}$ and $\overline{PK'}$ have side length $\frac{v}{t}(t) = v$ and the segments $\overline{PJ'}$ and $\overline{PK'}$ lie on lines passing through the center of dilation. So, the dilation maps $K'$ to $N$ and $J'$ to $M$. A similarity transformation maps $\triangle JKL$ to $\triangle MNP$. So, $\triangle JKL$ is similar to $\triangle MNP$.

4.6 **Exercises (pp. 219–220)**

**Vocabulary and Core Concept Check**

1. Congruent figures have the same size and shape. Similar figures have the same shape, but not necessarily the same size.

2. A transformation that produces a similar figure, such as a dilation, is called a similarity transformation.

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3. Translation $(x, y) \rightarrow (x + 3, y + 1)$; $F(-2, 2) \rightarrow F'(1, 3), \quad G(-2, -4) \rightarrow G'(1, -3), \quad H(-4, -4) \rightarrow H'(1, -3)$

   Dilation $(x, y) \rightarrow (2x, 2y)$; $F'(1, 3) \rightarrow F''(2, 6), \quad G'(1, -3) \rightarrow G''(2, -6), \quad H'(1, -3) \rightarrow H''(-2, -6)$

4. Dilation $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$; $F(-2, 2) \rightarrow F'(-1, 1), \quad G(-2, -4) \rightarrow G'(-1, -2), \quad H(-4, -4) \rightarrow H'(-2, -2)$

   Reflection in the $y$-axis: $F'(-1, 1) \rightarrow F''(1, 1), \quad G'(-1, -2) \rightarrow G''(1, -2), \quad H'(-2, -2) \rightarrow H''(2, -2)$

5. Rotation $90^\circ$ about the origin: $F(-2, 2) \rightarrow F'(-2, -2), \quad G(-2, -4) \rightarrow G'(4, -2), \quad H(-4, -4) \rightarrow H'(4, -4)$

   Dilation $(x, y) \rightarrow (3x, 3y)$; $F'(-2, -2) \rightarrow F''(-6, -6), \quad G'(4, -2) \rightarrow G''(12, -6), \quad H'(4, -4) \rightarrow H''(12, -12)$
6. Dilation \((x, y) \rightarrow \left(\frac{3}{2}x, \frac{3}{2}y\right)\):
   \(F(-2, 2) \rightarrow F'(-1.5, 1.5)\),
   \(G(-2, -4) \rightarrow G'(-1.5, -3)\),
   \(H(-4, -4) \rightarrow H'(-3, -3)\)
   Reflection in the \(x\)-axis: \(F'(-1.5, 1.5) \rightarrow F''(-1.5, -1.5)\),
   \(G'(-1.5, -3) \rightarrow G''(-1.5, 3)\),
   \(H'(-3, -3) \rightarrow H''(-3, 3)\)

7. Sample answer: The similarity transformation is a translation 1 unit down and 1 unit right, and then a dilation with center at \(E'(2, -3)\) and a scale factor of 2.
   Translation \((x, y) \rightarrow (x + 1, y - 1)\):
   \(D(-3, -2) \rightarrow D'(-2, -3)\), \(E(1, -2) \rightarrow E'(2, -3)\),
   \(F(1, 0) \rightarrow F'(2, -1)\)
   Dilation: \(D'(-2, -3) \rightarrow T(-6, -3)\), \(E'(2, -3) \rightarrow U(2, -3)\),
   \(F'(2, -1) \rightarrow V(2, 1)\)

8. Sample answer: The similarity transformation is a dilation with center at the origin and a scale factor of \(\frac{1}{2}\), and then a reflection in the \(y\)-axis.
   Dilation with a scale factor of \(\frac{1}{2}\): \(L(-2, 8) \rightarrow L'(-1, 4)\),
   \(K(6, 6) \rightarrow K'(3, 3)\), \(J(6, 2) \rightarrow J'(3, 1)\), \(M(0, 2) \rightarrow M'(0, 1)\)
   Reflection in the \(y\)-axis: \(L'(-1, 4) \rightarrow R(1, 4)\),
   \(K'(3, 3) \rightarrow Q(-3, 3)\), \(J'(3, 1) \rightarrow P(-3, 1)\), \(M'(0, 1) \rightarrow S(0, 1)\)

9. yes; Sample answer: \(\triangle ABC\) can be mapped to \(\triangle DEF\) by a dilation with center at the origin and a scale factor of \(\frac{1}{3}\), followed by a translation of 2 units left and 3 units up.

10. yes; Sample answer: \(\square QRS T\) can be mapped to \(\square WXYZ\) by a \(270^\circ\) rotation about the origin followed by a dilation with center at the origin and a scale factor of 2.

11. no; Side lengths are not proportional. The scale factor from \(HI\) to \(JL\) is \(\frac{3}{5}\), but the scale factor from \(GH\) to \(KL\) is \(\frac{5}{3}\).

12. no; Side lengths are not proportional. The scale factor from \(DG\) to \(LP\) is 1, but the scale factor from \(FG\) to \(NP\) is \(\frac{5}{3}\).

13. Given: Right isosceles \(\triangle ABC\), with leg length \(j\)
   Right isosceles \(\triangle RST\), with leg length \(k\)
   \(CA || RT\)
   Prove: \(\triangle ABC\) is similar to \(\triangle RST\).
   Reflect \(\triangle ABC\) in \(AB\). Because reflections preserve side lengths and angle measures, the image of \(\triangle ABC\), \(\triangle ABC'\),
   is a right isosceles triangle with leg length \(j\). Also because \(\overline{AC} \perp \overline{BA}\), point \(C'\) is on \(\overline{AC}\). So, \(\overline{AC'}\) is parallel to \(\overline{RT}\).

   Then translate \(\triangle ABC'\) so that point \(A\) maps to point \(R\).
   Because translation map segments to parallel segments and \(\overline{AC'} || \overline{RT}\), the image of \(\overline{AC'}\) lies on \(\overline{RT}\).

   Because translations preserve side lengths and angle measures, the image of \(\triangle ABC'\), \(\triangle RB'C'\), is a right isosceles triangle with leg length \(j\). Because \(\angle RB'C'\) and \(\angle RST\) are right angles, they are congruent. When \(\overline{RC'}\) coincides with \(\overline{RT}\), \(\overline{RB'}\) coincides with \(\overline{RS}\). So, \(\overline{RB'}\) lies on \(\overline{RS}\). Next, dilate \(\triangle RB'C'\) using center of dilation \(R\). Choose the scale factor to be the ratio of the side lengths of \(\triangle RST\) and \(\triangle RB'C'\), which is \(\frac{k}{j}\).

   The dilations maps \(\overline{RC''}\) to \(\overline{RT}\) and \(\overline{RB}\) to \(\overline{RS}\) because the images of \(\overline{RC''}\) and \(\overline{RB}\) have side length \(\frac{k}{j}\) and the segments \(\overline{RC''}\) and \(\overline{RB}\) lie on lines passing through the center of dilation. So, the dilation maps \(C''\) to \(T\) and \(B'\) to \(S\). A similarity transformation maps \(\triangle ABC\) to \(\triangle RST\). So, \(\triangle ABC\) is similar to \(\triangle RST\).
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14. Given: Rectangle JKLM, with side lengths x and y
   Rectangle QRSST, with side lengths 2x and 2y
   Prove: Rectangle JKLM is similar to rectangle QRSST.
   If necessary, rotate □JKLM about point M so that ML′ || TS.

\[ \text{Note that because rotations are rigid motions, } \square J'K'L'M \text{ is congruent to } \square JKLM. \text{ Translate } \square J'K'L'M \text{ so that point } M \text{ maps to point } T. \text{ Because translations map segments to parallel segments and } ML' \parallel TS, \text{ the image of } ML' \text{ lies on } TS. \]

\[ J' \quad K' \quad M \quad L' \quad T' \quad T \]

\[ J \quad K \quad M \quad L \quad T \]

Because translations preserve side lengths and angle measures, the image of □J′K′L′M, □J′K′L′T, is a rectangle with side lengths x and y. Because all interior angles of a rectangle are right angles, ∠J′TL′ and ∠QTS are congruent. When TL′ coincides with TS, TJ′ coincides with TQ. So, TJ′ lies on TQ. Next, dilate □J′K′L′T using center of dilation T. Choose the scale factor to be the ratio of the side lengths of □QRSST and □J′K′L′T, which is

\[ \frac{2x}{x} = \frac{2}{1} = 2. \]

\[ \text{The dilation maps } TL' \text{ to } TS \text{ and } TJ' \text{ to } TQ \text{ because the images of } TL' \text{ and } TJ' \text{ have side lengths } y(2) = 2y \text{ and } x(2) = 2x, \text{ and the segments } TL' \text{ and } TJ' \text{ lie on lines passing through the center of dilation. So, the dilation maps } L' \text{ to } S \text{ and } J' \text{ to } Q. \text{ The image of } K' \text{ lies } y(2) = 2y \text{ units to the right of the image of } J', \text{ and } x(2) = 2x \text{ units above the image of } L'. \text{ So, the image of } K' \text{ is } R. \text{ A similarity transformation maps } \square JKLM \text{ to } \square QRSST. \text{ So, } \square JKLM \text{ is similar to } \square QRSST. \]

15. The stop sign sticker can be mapped to the regular-sized stop sign by translating the sticker to the left until the centers match, and then dilating the sticker with a scale factor of 3.15. Because there is a similarity transformation that maps one stop sign to the other, the sticker is similar to the regular-sized stop sign.

16. Figure A is not similar to Figure B because the scale factor (A to B) of the shorter legs is \( \frac{1}{2} \), and the scale factor (A to B) of the longer legs is \( \frac{2}{3} \).
   
   \[ \text{Sample answer:} \]

17. no; The scale factor is 6 for both dimensions. So, the enlarged banner is proportional to the smaller one.

18. a. yes; The suns appear to be a dilation of one another.
   b. no; The hearts are about the same height but one is wider than the other.

19. \[ \text{Sample answer: Let } k = 3. \text{ Then } (x, y) \rightarrow (3x, 3y) \text{ will map } \triangle ABC \text{ to } \triangle A'B'C'. \text{ Apply the translation } (x, y) \rightarrow (x + 2, y - 3) \text{ to map } \triangle A'B'C' \text{ to } \triangle A''B''C''. \]
   
   \[ \text{Apply the reduction dilation } (x, y) \rightarrow \left( \frac{1}{3}x, \frac{1}{3}y \right) \text{ and the translation } (x, y) \rightarrow (x - \frac{1}{3}, y + 1) \text{ to } \triangle A'B'C' \text{ to map } \triangle A''B''C'' \text{ to } \triangle ABC. \]

20. sometimes; As long as the center of dilation and the center of rotation are the same, rotations and dilations are commutative.

21. The vertices of JKLM are J(−8, 0), K(−8, 12), L(−4, 12), and M(−4, 0). The vertices of J′K′L′M′ are J′(−9, −4), K′(−9, 14), L′(−3, 14), and M′(−3, −4).
   
   \[ \text{JKLM} \sim \triangle J′K′L′M′; \text{ A similarity transformation mapped quadrilateral JKLM to quadrilateral J′K′L′M′.} \]

22. a. yes; \[ \text{Sample answer: This triangle can be mapped to the larger one by a } 180° \text{ rotation about the origin, followed by the translation } (x, y) \rightarrow (x + 5, y + 4), \text{ followed by a dilation with center } (1, 1) \text{ and a scale factor of } 2. \]
   
   \[ \text{Because one can be mapped to the other by a similarity transformation, the triangles are similar.} \]
   
   b. \[ \text{Sample answer: The triangle formed when the midpoints of a triangle are connected is always similar to the original triangle.} \]
Chapter 4

Maintaining Mathematical Proficiency

23. The angle with measure of 113° is an obtuse angle.

24. The angle is a line, so it is a straight angle.

25. The angle with measure of 82° is an acute angle.

26. The angle is a right angle.

4.4–4.6 What Did You Learn? (p. 221)

1. Sample answer: Draw a picture and label the given information. Then look at the results and try to figure out what needs to be proven in order to get there. When unsure look back at related definitions, postulates, and theorems to see which ones might be helpful. Points L and M must be identified, so use the Ruler Postulate (Post. 1.1) and Segment Addition Postulate (Post 1.2). Then the rest will start falling into place.

2. Sample answer: This drawing could represent the reduction of a 16 × 28 painting into a 4 × 7 photograph or computer graphic.

Chapter 4 Review (pp. 222–224)

1. \((x, y) \rightarrow (x, y + 2)\)
   \(X(2, 3) \rightarrow X'(2, 5)\)
   \(Y(-3, 2) \rightarrow Y'(-3, 4)\)
   \(Z(-4, -3) \rightarrow Z'(-4, -1)\)

2. \((x, y) \rightarrow (x - 3, y)\)
   \(X(2, 3) \rightarrow X'(-1, 3)\)
   \(Y(-3, 2) \rightarrow Y'(-6, 2)\)
   \(Z(-4, -3) \rightarrow Z'(-7, -3)\)

3. \((x, y) \rightarrow (x + 3, y - 1)\)
   \(X(2, 3) \rightarrow X'(5, 2)\)
   \(Y(-3, 2) \rightarrow Y'(0, 1)\)
   \(Z(-4, -3) \rightarrow Z'(-1, -4)\)

4. \((x, y) \rightarrow (x + 4, y + 1)\)
   \(X(2, 3) \rightarrow X'(6, 4)\)
   \(Y(-3, 2) \rightarrow Y'(1, 3)\)
   \(Z(-4, -3) \rightarrow Z'(0, -2)\)

5. The image of \(\triangle PQR\) after the translation \((x, y) \rightarrow (x + 1, y + 2)\) will have the vertices \(P'(1, -2), Q'(2, 5)\), and \(R'(3, -3)\). After the translation \((x, y) \rightarrow (x - 4, y + 1)\), the vertices are \(P''(-3, -1)\), \(Q''(-2, 6)\), and \(R''(-1, -2)\).
6. The image of \( \triangle PQR \) after the translation \((x, y) \rightarrow (x, y + 3)\) will have the vertices \(P'(0, -1), \ Q'(1, 6), \) and \(R'(2, -2)\). After the translation \((x, y) \rightarrow (x - 1, y + 1)\), the vertices are \(P''(-1, 0), \ Q''(0, 7), \) and \(R''(1, -1)\).

7. Reflection in \(x = 4\):
   \(A(1, 2) \rightarrow A'(7, 2)\)
   \(B(3, 4) \rightarrow B'(5, 4)\)
   \(C(5, 1) \rightarrow C'(3, 1)\)

8. Reflection in \(y = 3\):
   \(E(1, 3) \rightarrow E'(1, 3)\)
   \(F(4, 3) \rightarrow F'(4, 3)\)
   \(G(5, 0) \rightarrow G'(5, 6)\)
   \(H(2, 0) \rightarrow H'(2, 6)\)

9. There are two lines of symmetry.

10. \((x, y) \rightarrow (-y, x)\)
    \(A(-3, -1) \rightarrow A'(1, -3)\)
    \(B(2, 2) \rightarrow B'(-2, 2)\)
    \(C(3, -3) \rightarrow C'(3, 3)\)

11. \((x, y) \rightarrow (-x, -y)\)
    \(W(-2, -1) \rightarrow W'(2, 1)\)
    \(X(-1, 3) \rightarrow X'(1, -3)\)
    \(Y(3, 3) \rightarrow Y'(-3, -3)\)
    \(Z(3, -3) \rightarrow Z'(-3, 3)\)

12. Reflection in \(x\)-axis \((x, y) \rightarrow (x, -y)\):
    \(X(5, -2) \rightarrow X'(5, 2)\)
    \(Y(3, -3) \rightarrow Y'(3, 3)\)
    Rotation of \(270^\circ\) about the origin \((x, y) \rightarrow (-y, x)\):
    \(X'(5, 2) \rightarrow X''(2, -5)\) and \(Y'(3, 3) \rightarrow Y''(3, -3)\)

13. yes; The rotations of \(60^\circ, 120^\circ,\) and \(180^\circ\) about the center will map this figure onto itself.

14. yes; The rotations of \(72^\circ\) and \(144^\circ\) about the center will map this figure onto itself.

15. **Sample answer:** The congruence transformation that maps \(\triangle DEF\) to \(\triangle JKL\) is a reflection in the \(y\)-axis, and then a translation 3 units down.
    Reflection: \(D(2, -1) \rightarrow D'(-2, -1)\)
    \(E(4, 1) \rightarrow E'(-4, 1)\)
    \(F(1, 2) \rightarrow F'(-1, 2)\)
    Translation: \(D'(-2, -1) \rightarrow J(-2, -4)\)
    \(E'(-4, 1) \rightarrow K(-4, -2)\)
    \(F'(-1, 2) \rightarrow L(-1, -1)\)

16. **Sample answer:** The congruence transformation that maps \(\triangle DEF\) to \(\triangle JKL\) is a reflection in the \(x\)-axis, and then a translation 4 units right.
    Reflection: \(D(-3, -4) \rightarrow D'(-3, 4)\)
    \(E(-5, -1) \rightarrow E'(-5, 1)\)
    \(F(-1, 1) \rightarrow F'(-1, -1)\)
    Translation: \(D'(-3, 4) \rightarrow J(1, 4)\)
    \(E'(-5, 1) \rightarrow K(-1, 1)\)
    \(F'(-1, -1) \rightarrow L(3, -1)\)

17. A translation is a reflection in two parallel lines. A rotation is a reflection in two intersecting lines.

18. \((x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)\)
    \(P(2, 2) \rightarrow P'(1, 1)\)
    \(Q(4, 4) \rightarrow Q'(2, 2)\)
    \(R(8, 2) \rightarrow R'(4, 1)\)

19. \((x, y) \rightarrow (-3x, -3y)\)
    \(X(-3, 2) \rightarrow X'(9, -6)\)
    \(Y(2, 3) \rightarrow Y'(-6, -9)\)
    \(Z(1, -1) \rightarrow Z'(-3, 3)\)
20. \(8x = 15.2\)
\[x = \frac{15.2}{8}\]
\[x = 1.9\]

The actual length of the object is 1.9 centimeters.

21. **Sample answer:** The similarity transformation that maps \(\triangle ABC\) to \(\triangle RST\) is a dilation with a scale factor of \(-3\) followed by a reflection in the \(x\)-axis.

Dilation: \(A(1, 0) \rightarrow A'(−3, 0), B(-2, −1) \rightarrow B'(6, 3), C(−1, −2) \rightarrow C'(3, 6)\)

Reflection: \(A'(−3, 0) \rightarrow R(−3, 0), B'(6, 3) \rightarrow S(6, −3), C'(3, 6) \rightarrow T(3, −6)\)

22. **Sample answer:** The similarity transformation that maps \(\triangle ABC\) to \(\triangle RST\) is a reflection in the line \(y = x\) followed by a dilation with a scale factor of \(\frac{1}{2}\).

Reflection: \(A(6, 4) \rightarrow A'(4, 6), B(-2, 0) \rightarrow B'(0, −2), C(−4, 2) \rightarrow C'(2, −4)\)

Dilation: \(A'(4, 6) \rightarrow R(2, 3), B'(0, −2) \rightarrow S(0, −1), C'(2, −4) \rightarrow T(1, −2)\)

23. **Sample answer:** The similarity transformation that maps \(\triangle ABC\) to \(\triangle RST\) is a rotation of \(270°\) about the origin followed by a dilation with a scale factor of \(2\).

Rotation: \(A(3, −2) \rightarrow A'(−2, −3), B(0, 4) \rightarrow B'(4, 0), C(−1, −3) \rightarrow C'(−3, 1)\)

Dilation: \(A'(−2, −3) \rightarrow R(−4, −6), B'(4, 0) \rightarrow S(8, 0), C'(−3, 1) \rightarrow T(−6, 2)\)

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**Chapter 4 Test (p. 225)**

1. \((x, y) \rightarrow (x − 4, y + 1)\)
   \[R(−4, 1) \rightarrow R'(−8, 2)\]
   \[S(−2, 2) \rightarrow S'(−6, 3)\]
   \[T(3, −2) \rightarrow T'(−1, −1)\]

2. \((x, y) \rightarrow (x + 2, y − 2)\)
   \[R(−4, 1) \rightarrow R'(−2, −1)\]
   \[S(−2, 2) \rightarrow S'(0, 0)\]
   \[T(3, −2) \rightarrow T'(5, −4)\]

3. \((x, y) \rightarrow (y, −x)\)
   \[D(−1, −1) \rightarrow D'(−1, 1)\]
   \[E(−3, 2) \rightarrow E'(2, 3)\]
   \[F(1, 4) \rightarrow F'(4, −1)\]

4. \((x, y) \rightarrow (−y, x)\)
   \[J(−1, 1) \rightarrow J'(−1, −1)\]
   \[K(3, 3) \rightarrow K'(−3, 3)\]
   \[L(4, −3) \rightarrow L'(3, 4)\]
   \[M(0, −2) \rightarrow M'(2, 0)\]

5. Quadrilateral \(QRST\) is similar to quadrilateral \(WXYZ\). **Sample answer:** Quadrilateral \(QRST\) can be mapped to quadrilateral \(WXYZ\) by a dilation with a scale factor of \(3\) and a reflection in the \(x\)-axis.

Dilation: \(Q(2, 4) \rightarrow Q'(6, 12)\)
   \[R(5, 4) \rightarrow R'(15, 12)\]
   \[S(6, 2) \rightarrow S'(18, 6)\]
   \[T(1, 2) \rightarrow T'(3, 6)\]

Reflection: \(Q'(6, 12) \rightarrow W(6, −12)\)
   \[R'(15, 12) \rightarrow X(15, −12)\]
   \[S'(18, 6) \rightarrow Y(18, −6)\]
   \[T'(3, 6) \rightarrow Z(3, −6)\]

Because this composition has a rigid motion and a dilation, it is a similarity transformation.

6. \(\triangle DEF\) is congruent to \(\triangle ABC\). **Sample answer:** \(\triangle ABC\) can be mapped to \(\triangle DEF\) by a rotation of \(270°\) about the origin and a translation \(1\) unit up and \(3\) units right.

Rotation: \(A(−6, 6) \rightarrow A'(6, 6)\)
   \[B(−6, 2) \rightarrow B'(2, 6)\]
   \[C(−2, −4) \rightarrow C'(−4, 2)\]

Translation: \(A'(6, 6) \rightarrow D(9, 7)\)
   \[B'(2, 6) \rightarrow E(5, 7)\]
   \[C'(−4, 2) \rightarrow F(−1, 3)\]

Because this is a composition of two rigid motions, the composition is rigid.

7. yes; yes; The lines of symmetry are vertically through the center of the ball and horizontally through the center of the ball; \(180°\) rotational symmetry.

8. yes; no; The line of symmetry runs from the center of the base of the guitar, and through the sound hole to the center of the headstock of the guitar.

9. no; yes; \(180°\) rotational symmetry.
Chapter 4

10. Parallelogram $ABCD$ is reflected in the line $y = -x + 3$ and $A'B'C'D'$ is reflected in the line $y = -x + 3$.

11. no; Each vertex has traded places with one other vertex.

12. Sample answer: A composition of transformations that maps $\triangle ABC$ onto $\triangle CDB$ is a reflection in the $x$-axis followed by the translation $(x, y) \rightarrow (x + 1, y + 2)$. Both reflections and translations are rigid motions. So, according to the Composition Theorem (Thm. 4.1), this composition is a congruence transformation.

13. a. Sample answer: The similarity transformation that maps slice $ABC$ to slice $DEF$ is a rotation $270^\circ$ about the origin followed by a dilation with center at the origin and $k = \frac{1}{2}$, followed by a translation $(x, y) \rightarrow (x + 1, y - 1)$.

Rotation: $A(0, 0) \rightarrow A'(0, 0), B(2, 4) \rightarrow B'(4, -2), C(4, 2) \rightarrow C'(2, -4)$

Dilation: $A'(0, 0) \rightarrow A''(0, 0), B'(4, -2) \rightarrow B''(2, -1), C'(2, -4) \rightarrow C''(1, -2)$

Translation: $A''(0, 0) \rightarrow D(1, -1), B''(2, -1) \rightarrow E(3, -2), C''(1, -2) \rightarrow F(2, -3)$

b. Sample answer: A medium slice would be between a small and a large, and $\frac{1}{2} < \frac{2}{3} < 1$. So, $k = \frac{3}{4}$.

14. a. To produce the new photograph, reflect, reduce (dilate), and translate the original photograph.

b. Width = 4 in. → $4 \times \frac{3}{4} = 2$

Length = 6 in. → $6 \times \frac{3}{4} = 3$

The new dimensions are 2 inches by 3 inches.

c. no; The scale factor for the shorter sides is $\frac{17}{8}$, but the scale factor for the longer sides is $\frac{15}{6}$. So, the photo would have to be cropped or distorted in order to fit the frame.

Chapter 4 Standards Assessment (pp. 226–227)

1. a. $(x, y) \rightarrow (x - 4, y - 3)$
   $A(-1, 2) \rightarrow A'(-5, -1)$
   $A'(-5, -1) \rightarrow D(1, -5)$
   $B(3, 4) \rightarrow B'(1, -1)$
   $B'(1, -1) \rightarrow E(-1, -1)$
   $C(2, 3) \rightarrow C'(-2, -1)$
   $C'(-2, -1) \rightarrow F(1, -2)$

2. Step 1. Place the compass at $P$. Draw an arc that intersects line $m$ in two different places. Label the points of intersection $A$ and $B$.

   Step 2. With the compass at $A$ draw an arc below line $m$ using a setting greater than $\frac{1}{2} AB$. Using the same compass setting, draw an arc from $B$ that intersects the previous arc. Label the intersection $Q$.

   Step 3. Use a straightedge to draw $PQ$.

3. yes; She could find the side lengths and the bottom length by counting units. Then use the Pythagorean Theorem or Distance Formula to find the lengths of the angled sides.

4. The point is $P(0, 1)$.

5. a. Sample answer: A reflection in the line $y = -x$ will map $WXYZ$ to $ABCD$.
   $(x, y) \rightarrow (-y, -x)$
   $W(-1, 4) \rightarrow A(-4, 1)$
   $X(2, 3) \rightarrow B(-3, -2)$
   $Y(1, 1) \rightarrow C(-1, -1)$
   $Z(-1, 2) \rightarrow D(-2, 1)$

   b. yes; Because a reflection is a rigid motion, which preserves side lengths and angle measurements, $WXYZ$ is congruent to $ABCD$.

6. D: The slope of the line parallel to $y = -\frac{1}{3}x + b$ is $-\frac{1}{3}$.

   $y = -\frac{1}{3}x + b$

   $3 = -\frac{1}{3}(-6) + b$

   $3 = 2 + b$

   $b = 1$

   So, the equation of the parallel line is $y = -\frac{1}{3}x + 1$.

7. A scale factor for a dilation of $AB$ that is shorter than $AB$ would be a reduction. The scale factor for a reduction is less than 1. So, the scale factors are $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{4}$.

8. a. One possible set of coordinates for quadrilateral $ABCD$, if reflected in the $y$-axis, that will map it onto itself is $A(-2, 2), B(2, 2), C(2, -2),$ and $D(-2, -2)$.

   b. One possible set of coordinates for quadrilateral $ABCD$, if reflected in the $x$-axis, that will map it onto itself is $A(-2, 2), B(2, 2), C(2, -2),$ and $D(-2, -2)$.

   c. One possible set of coordinates for quadrilateral $ABCD$, if rotated $90^\circ$ about the origin, that will map it onto itself is $A(-2, 2), B(2, 2), C(2, -2),$ and $D(-2, -2)$.

   d. One possible set of coordinates for quadrilateral $ABCD$, if rotated $270^\circ$ about the origin, that will map it onto itself is $A(-2, 2), B(2, 2), C(2, -2),$ and $D(-2, -2)$.